

# Dynamical Systems and Solar Sails

Ariadna Farrés and Àngel Jorba  
Departament de Matemàtica Aplicada i Anàlisi  
Universitat de Barcelona  
Gran Via 585, 08007 Barcelona, Spain  
E-mails: [ariadna.farres@maia.ub.es](mailto:ariadna.farres@maia.ub.es), [angel@maia.ub.es](mailto:angel@maia.ub.es)

Dynamical systems have proven to be a useful tool for the design of space missions. For instance, the use of invariant manifolds is now common to design transfer strategies.

Solar Sailing is a proposed form of spacecraft propulsion, where large membrane mirrors take advantage of the solar radiation pressure to push the spacecraft. Although the acceleration produced by the radiation pressure is smaller than the one achieved by a traditional spacecraft it is continuous and unlimited. This makes some long term missions more accessible, and opens a wide new range of possible applications that cannot be achieved by a traditional spacecraft. The effect of the sail depends on the sail lightness number ( $\beta$ ), which measures the efficiency of the sail, and two angles ( $\alpha$  and  $\delta$ ) that define the sail orientation.

In this talk we will focus on the dynamics of a Solar sail in several situations. We will introduce this problem focusing on a solar sail in the Earth-Sun system. In this case, the model used will be the Restricted Three Body Problem (RTBP) plus Solar radiation pressure. For a fixed value of  $\beta$ , the effect of the solar radiation pressure on the RTBP produces a 2D family of “artificial” equilibria, that can be parametrised by  $\alpha$  and  $\delta$ .

We will describe the dynamics around some of these “artificial” equilibrium points. We note that, due to the solar radiation pressure, the system is Hamiltonian only for two cases: when the sail is perpendicular to the Sun - Sail line; and when the sail is aligned with the Sun - sail line (i.e., no sail effect). The main tool used to understand the dynamics will be the computation of centre manifolds, both for Hamiltonian and non-Hamiltonian cases. Then, we will see how to use the stable and unstable manifolds to design station keeping strategies for the sail. The main idea is to understand how these invariant manifolds depend on the sail orientation. Then, when the sail is too far from the target point, we change the sail orientation so that the natural dynamics of the system brings it back to the point. The same strategy can be used around periodic orbits.

The second example is the dynamics of a Solar sail close to an asteroid. Note that, in this case, the effect of the sail becomes very relevant due to the low mass of the asteroid. We will use, as a model, a Hill problem plus the effect of the Solar radiation pressure, and we will describe some aspects of the natural dynamics of the sail.