

CONFERENCE ON QUALITATIVE AND GEOMETRIC ASPECTS OF ELLIPTIC PDE'S



September 2 to 6, 2013

**Centre de Recerca Matemàtica
Bellaterra, Barcelona**

Speakers

Vicent Caselles, *Universitat Pompeu Fabra, Barcelona, Spain*
Manuel del Pino, *Universidad de Chile, Santiago, Chile*
Alberto Farina, *Université de Picardie Jules Verne, Amiens, France*
Alessio Figalli, *The University of Texas, Austin, USA*
François Hamel, *IUF, Marseille, France*
Nikola Kamburov, *MIT, Cambridge, USA*
Moritz Kassmann, *Universität Bielefeld, Germany*
Enno Lenzmann, *University of Copenhagen, Denmark*
Yanyan Li, *Rutgers University, New Brunswick, USA*
Chang-Shou Lin, *National Taiwan University, Taipei, Taiwan*
Carlo Mantegazza, *Scuola Normale Superiore di Pisa, Italy*
Robert McCann, *University of Toronto Bahen Centre, Toronto, Canada*
Frank Morgan, *Williams College, Massachusetts, USA*
Nikolai Nadirashvili, *CNRS et Université d'Aix-Marseille, France*
Frank Pacard, *École Polytechnique and Université Paris Est-Créteil, Paris, France*
Ireneo Peral, *Universidad Autónoma de Madrid, Spain*
Joaquin Pérez, *Universidad de Granada, Spain*
Aldo Pratelli, *Universität Erlangen-Nürnberg, Germany*
Jean-Michel Roquejoffre, *Université Paul Sabatier, Toulouse, France*
Antonio Ros, *Universidad de Granada, Spain*
Bernardino Sciunzi, *Università della Calabria, Cosenza, Italy*
Joel Spruck, *John Hopkins University, Baltimore, USA*
Gabriella Tarantello, *Università di Roma "Tor Vergata", Italy*
Juan Luis Vázquez, *Universidad Autónoma de Madrid, Spain*
Jun-Cheng Wei, *Chinese University of Hong Kong, Shatin, China*

Organising and Scientific Committee

Xavier Cabré, *ICREA and Universitat Politècnica de Catalunya*
Daniele Castorina, *Università di Roma "Tor Vergata"*
Manel Sanchón, *Universitat de Barcelona*
Enrico Valdinoci, *Università di Milano and Weierstrass Institute for Applied Analysis and Stochastics*

Deadline for abstracts submission: **June 30, 2013**
Deadline for registration and payment: **July 7, 2013**

www.crm.cat/2013/CELLipticPDEs



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1. PRACTICAL INFORMATION

You can check the updated programme and the list of participants at:
<http://www.crm.cat/en/Activities/Documents/programa.pdf>

Registration: Registration to the activity will take place at the CRM, located in the Science Building (Edifici de Ciències), Universitat Autònoma de Barcelona in Bellaterra. You can check our location at: <http://www.crm.cat/en/AboutTheCRM/Pages/LocationDirections.aspx>

Lecture room: The activity will take place in the CRM Auditorium, located in the Sciences Building (Edifici de Ciències), Universitat Autònoma de Barcelona in Bellaterra.

Activity documents: Attendance certificates and registration fee receipts will be available on demand. A check list will be posted on the activities board for that purpose.

Participants can request the documents until **Wednesday**. The documents will be available to be picked up at the activities coordinator desk on **Friday morning**.

*Invoice: in case you need an invoice with the details of your university, please send me an email asking for it (include the information you need to be written on it).

Secretariat: The Secretariat of the CRM will be available to the participants Monday through Friday from 9:30 am to 13:30 pm.

SAF (gym at the UAB–Servei d'Activitat Física): If you have booked an apartment at the Vila Universitària we inform you that you can have free access to the SAF. Please ask at the Vila reception desk to know the conditions. They will then prepare a certificate for you in order to have free access.

Computer facilities: The computer space of the CRM will be available for the participants of the conference.

The timetable is Monday through Friday from 8:30 am to 6:00 pm. The CRM premises as well as most of the UAB campus have wireless access.

Wifi password: crmwifikey

Library: The library of the Science Building of the UAB will be open from 8:30 am to 7:30 pm on working days.

Breaks: Coffee and cookies will be served during the morning breaks to all participants.

Social events: We have organized a guided visit to Barcelona and a social dinner for Wednesday afternoon. Detailed information will be available on the activity's webpage. Registration will be necessary before Tuesday September 3rd at noon by signing a document which will be at the auditorium's entrance door for that purpose.

Picture: A group picture will be taken on Wednesday, September 4th before the coffee break. We will inform you of the place to meet. The picture will be posted on the activity's webpage.

Questionnaire: Following the directions of the CRM Governing Board, we give a questionnaire to all the people participating in activities at the CRM in order to assess their level of satisfaction. The questionnaire is anonymous and not mandatory, but we would greatly appreciate it if you could answer the questions and return it to us. Thank you for your cooperation.

Local emergency numbers: General emergency (police, ambulance, fire-fighters) call 112.

Safety in Barcelona: Although Barcelona is a safe city, please be aware that there is a problem with pickpockets, especially around tourist areas: La Rambla, Plaça Catalunya, Barcelona Airport, major metro and train stations, famous buildings, etc. Be sure to keep your belongings with you at all times, be alert, and be wary of unusual situations.

2. SCHEDULE OF THE WORKSHOP

Monday, September 2	
09:15 – 09:30	REGISTRATION
09:30 – 10:25	Juan Luis Vázquez <i>The theory of fractional heat and porous medium equations</i>
10:30 – 11:00	COFFEE BREAK
11:00 – 11:55	Jean-Michel Roquejoffre
12:05 – 13:00	Manuel del Pino <i>Some examples of nonlocal minimal surfaces</i>
13:00 – 14:30	LUNCH
14:30 – 15:25	Gabriella Tarantello <i>The role of singular Liouville systems in the study of non-abelian Chern-Simons vortices</i>
15:30 – 15:50	COFFEE BREAK
15:50 – 16:45	Enno Lenzmann <i>ODE-type Results for the fractional Laplacian</i>
16:55 – 17:50	Nikola Kamburov <i>A free boundary variant of a conjecture of De Giorgi</i>
Tuesday, September 3	
09:30 – 10:25	Frank Morgan <i>The log-convex density conjecture</i>
10:30 – 11:00	COFFEE BREAK
11:00 – 11:55	Nikolai Nadirashvili <i>Nonlinear elliptic equations and geometry of flow of ideal fluid</i>
12:05 – 13:00	Robert McCann <i>Hölder continuity of maps optimizing MTW costs</i>
13:00 – 14:30	LUNCH
14:30 – 15:25	François Hamel <i>Semilinear elliptic equations in convex domains and convex rings</i>
15:30 – 15:50	COFFEE BREAK
15:50 – 16:45	Alberto Farina <i>One-dimensional symmetry for entire solutions of an elliptic system arising in phase separation</i>
16:55 – 17:50	Moritz Kassmann <i>The Dirichlet problem for nonlocal operators</i>

Wednesday, September 4	
09:30 – 10:25	Ireneo Peral <i>Some inequalities for the fractional laplacian: elliptic and parabolic equations</i>
10:30 – 11:00	COFFEE BREAK
11:00 – 11:55	Joaquin Pérez <i>Quadratic decay of curvature on minimal surfaces</i>
12:05 – 13:00	Alessio Figalli <i>Partial regularity for Monge-Ampère type equations</i>
13:00 – 14:30	LUNCH
Thursday, September 5	
09:30 – 10:25	Chang-Shou Lin <i>On the exact number of solutions of Chern-Simons-Higgs equation with double vortices</i>
10:30 – 11:00	COFFEE BREAK
11:00 – 11:55	Carlo Mantegazza <i>Locally conformally flat non-gradient Ricci solitons</i>
12:05 – 13:00	Jun-Cheng Wei <i>Monotonicity formula and applications to supercritical elliptic problems</i>
13:00 – 14:30	LUNCH
14:30 – 15:25	Bernardino Sciunzi <i>Blow-up for the nonlinear heat equation</i>
15:30 – 15:50	COFFEE BREAK
15:50 – 16:45	Frank Pacard
Friday, September 6	
09:30 – 10:25	A Ros
10:30 – 11:00	COFFEE BREAK
11:00 – 11:55	A Pratelli <i>On the existence of isoperimetric sets in R^N with density</i>
12:05 – 13:00	
13:00 – 14:30	LUNCH

3. ABSTRACTS OF THE SPEAKERS

Manuel del Pino

Some examples of nonlocal minimal surfaces.

Abstract: We will analyze stability and instability for s -minimal cones and present examples of global minimal surfaces of revolution, including the analogue to the catenoid when s approaches the limiting value in which s -minimal surfaces become minimal in the usual sense.

Contact address: ma.delpino@yahoo.com

Alberto Farina

One-dimensional symmetry for entire solutions of an elliptic system arising in phase separation.

Abstract: We consider positive solutions of the elliptic system

$$\begin{cases} \Delta u = uv^2 & \text{in } \mathbb{R}^N \\ \Delta u = vu^2 & \text{in } \mathbb{R}^N, \end{cases}$$

for every dimension $N \geq 2$.

I will present new one-dimensional symmetry results for solutions with at most algebraic growth at infinity.

Contact address: alberto.farina@u-picardie.fr

Alessio Figalli

Partial regularity for Monge-Ampère type equations.

Abstract: Monge-Ampère type equations arise in several problems from analysis and geometry, and understanding their regularity is an important question. In particular, this kind of equations arises in the regularity theory of optimal transport maps.

In the 90's Caffarelli developed a regularity theory on R^n for the classical Monge-Ampère equation, which was then extended by Ma-Trudinger-Wang and Loeper to a more general class of equations which satisfy a suitable structural condition. Unfortunately, this condition is very restrictive and it is satisfied only in very particular cases. Hence the need to develop a partial regularity theory: is it true that optimal maps are always smooth outside a “small” singular set? The aim of this talk is first to review the “classical” regularity theory for optimal maps, and then to describe some recent results about their partial regularity.

Contact address: figalli@math.utexas.edu

François Hamel***Semilinear elliptic equations in convex domains and convex rings.***

Abstract: In this talk, I will discuss some geometrical properties of positive solutions of some semilinear elliptic equations in bounded convex domains or convex rings, with Dirichlet-type boundary conditions. A solution is called quasiconcave if its superlevel sets are convex. I will present two counterexamples, that is two cases of semilinear elliptic equations for which the solutions are not quasiconcave.

This is a joint work with N. Nadirashvili and Y. Sire.

Contact address: francois.hamel@univ-amu.fr

Nikola Kamburov***A free boundary variant of a conjecture of De Giorgi.***

Abstract: Drawing out a fascinating connection between Bernstein's problem, on the one hand, and the study of global, bounded, and monotone solutions to the semilinear elliptic equation $\Delta u = u^3 - u$ in \mathbb{R}^n , on the other, a famous conjecture of De Giorgi states that the level sets of such solutions are hyperplanes, at least in dimension $n \leq 8$. The conjecture was verified for $n \leq 8$ by Savin. Recently, Del Pino, Kowalczyk, and Wei constructed a counterexample in dimension $n = 9$, using an intricate fixed point argument. I would like to discuss the construction of such a counterexample in an appealing free boundary variant of De Giorgi's conjecture. Our approach uses only the elementary means of the method of barriers.

Contact address: kamburov@math.arizona.edu

Moritz Kassmann***The Dirichlet problem for nonlocal operators.***

Abstract: In the talk we set up the Dirichlet problem for symmetric and non-symmetric nonlocal operators. We show existence of solutions with the help of classical tools like the Fredholm alternative

Contact address: moritz.kassmann@uni-bielefeld.de

Enno Lenzmann***ODE-type Results for the fractional Laplacian.***

Abstract: I will review recent joint work with R. Frank and L. Silvestre on linear and nonlinear elliptic radial problems with the fractional Laplacian. In particular, we show nondegeneracy and uniqueness of nonlinear ground states for the fractional Laplacian in any dimension. The proof combines three main ideas: 1.) topological bounds by half-space extensions, 2.) continuity arguments in the

power of the fractional Laplacian, and 3.) a local monotonicity formula inspired by the recent work of Cabré and Sire on layer solutions.

Contact address: enno.lenzmann@unibas.ch

Chang-Shou Lin

On the exact number of solutions of Chern-Simons-Higgs equation with double vortices.

Abstract: In this lecture I will prove the uniqueness of the bubbling solutions of the Chern-Simons-Higgs equation with double vortices. As a corollary of our result, we can prove the exact number of solutions for small parameter.

Contact address: cslin@math.ntu.edu.tw

Carlo Mantegazza

Locally conformally flat non-gradient Ricci solitons.

Abstract: I will discuss the structure of the general non-gradient Ricci solitons, that is, self-similar solutions of the Ricci flow, under the assumption that they are locally conformally flat. The conclusion is that (forgetting the Euclidean factors) they share a rotational symmetry, indeed, they are warped products of spheres on real intervals.

Contact address: c.mantegazza@sns.it

Robert McCann

Hölder continuity of maps optimizing MTW costs.

Abstract: In this talk we sketch recent progress on the regularity theory of optimal mappings obtained in joint work with Alessio Figalli and Young-Heon Kim.

Consider transportation of one distribution of mass onto another, chosen to optimize the total expected cost, where cost per unit mass transported from X to Y is given by a smooth function $c(X, Y)$. If the source density $f^+(X)$ is bounded away from zero and infinity in an open region $U \subset R^n$, and the target density $f^-(Y)$ is bounded away from zero and infinity on its support $V \subset R^n$, which is strongly c -convex with respect to U , and the transportation cost c satisfies the (A3w) condition of Ma, Trudinger and Wang, we deduce Hölder continuity and injectivity of the optimal map inside U . This result provides a crucial missing step in the low/interior regularity setting: in subsequent work, we use it to establish regularity of optimal maps with respect to the Riemannian distance squared on arbitrary products of spheres.

Connections to geometry and economics may be mentioned, if time permits.

Contact address: mccann@math.toronto.edu

Frank Morgan

The log-convex density conjecture.

Abstract: The log-convex density conjecture says that in \mathbb{R}^n with a log-convex radial density, balls about the origin minimize weighted perimeter for given weighted volume. We'll discuss some recent progress and generalizations following earlier work by undergraduates (see arXiv posts by Howe, Cañete/Rosales, Cabré/Ros-Oton/Serra).

Contact address: frank.morgan@williams.edu

Nikolai Nadirashvili

Nonlinear elliptic equations and geometry of flow of ideal fluid.

Abstract: We study nodal properties of solutions to quasilinear elliptic equations and consider its application to the geometry of streamlines of ideal fluid.

Contact address: nicolas@cmi.univ-mrs.fr

Ireneo Peral

Some inequalities for the fractional laplacian: elliptic and parabolic equations.

Abstract: We will study some inequalities involving $(-\Delta)^s$.

A review of the summability of the solutions to

$$\begin{cases} (-\Delta)^s u = f & \text{in } \Omega \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega \end{cases}$$

according with the summability of the data will be obtained by nonlinear methods.

Also the heat equation with fractional diffusion will be considered.

Contact address: ireneo.peral@uam.es

Joaquin Pérez

Quadratic decay of curvature on minimal surfaces.

Abstract: For a surface in Euclidean space \mathbb{R}^3 , the function that measures its absolute Gaussian curvature times the squared extrinsic distance to a given point p_0 , is invariant under homotheties centered at p_0 . When this function is bounded, the Gaussian curvature decays at most quadratically with respect to the extrinsic distance to p_0 . In this talk we will deduce strong geometric properties for embedded minimal surfaces with quadratic decay of curvature (QDC). The scale invariance nature of this assumption produces both local results (a removable singularity theorem for properly embedded minimal surfaces in a punctured ball

with QDC) and global results (the characterization of the complete embedded minimal surfaces in \mathbb{R}^3 with QDC as those having finite total curvature).

Contact address: jperez@ugr.es

Aldo Pratelli

On the existence of isoperimetric sets in R^N with density.

Abstract: We will consider the isoperimetric problem in the space R^N with density, that is, area and perimeter of sets are weighted with a given function. We will review the classical and the recent results about this problem (and also about the related questions of regularity and of boundedness of isoperimetric sets), and then we will focus on the main problem, that is, the existence of isoperimetric sets in the case when the density is approaching a finite limite at infinity from below.

This is a joint work with G. De Philippis and G. Franzina.

Contact address: aldo.pratelli@unipv.it

Bernardino Sciunzi

Blow-up for the nonlinear heat equation.

Abstract: Consider the nonlinear heat equation

$$(NLH) \quad v_t - \Delta v = |v|^{p-1}v,$$

in a domain $\Omega \subset \mathbb{R}^n$ and let \mathcal{G} be the set of initial data for which the corresponding solution is global. Considering the case of positive initial data it follows, under suitable assumptions, that \mathcal{G} is star-shaped. If else we consider changing sign initial data then, generally, \mathcal{G} is not star-shaped. This has been pointed out by T. Cazenave, F. Dickstein and F.B. Weissler in the case $n \geq 3$ and in the case when the domain is a ball in \mathbb{R}^n . In a first paper with F. Dickstein and F. Pacella we extend the result up to dimension two. Also in collaboration with V. Marino and F. Pacella we show that the same instability phenomenon occurs (in the case $n \geq 3$) in general domains.

Contact address: sciunzi@mat.unical.it

Gabriella Tarantello

The role of singular Liouville systems in the study of non-abelian Chern-Simons vortices.

Abstract: We describe recent results about the construction on non-abelian Chern-Simons vortices of non-topological type in terms of entire solutions for a class of singular Liouville systems in the plane.

Contact address: tarantel@mat.uniroma2.it

Juan Luis Vázquez

The theory of fractional heat and porous medium equations.

Abstract: We will report on recent research in the area of elliptic and parabolic equations of diffusion type where the Laplace operator is replaced by a fractional Laplacian operator, or other similar nonlocal operators.

The lecture will describe recent progress made by the author and collaborators on the topic of nonlinear fractional heat equations, in particular when the nonlinearities are of porous medium and fast diffusion type. We prove existence and uniqueness of solutions, regularity, positivity, Harnack estimates and symmetrization. Special attention is given to the construction of fractional Barenblatt solutions. We also study KPP phenomena.

This is a joint work with M. Bonforte, A. de Pablo, F. Quirós, A. Rodríguez, D. Stan, F del Teso, B. Volzone

Contact address: juanluis.vazquez@uam.es

Jun-Cheng Wei

Monotonicity formula and applications to supercritical elliptic problems.

Abstract: We first derive a monotonicity formula for fourth order supercritical Problem

$$\Delta^2 u = |u|^{p-1}u \text{ in } R^N, \quad p > (N+4)/(N-4)$$

and then based this monotonicity formula we give a complete and optimal classification of stable or finite Morse index solutions of positive or sign-changing solutions. Our approach is free of Moser's iterations and is based on tangent cone analysis of minimal surfaces. We also use the monotonicity formula to obtain optimal C^α regularity and convergence of sequence of positive solutions of negative exponent problem

$$\Delta u = u^{-p}, \quad p > 1.$$

Finally we use monotonicity formula to give a rather complete characterizations and analysis of blow-up locus for singular solutions of

$$\Delta u + u^p = 0, \quad p > (N+2)/(N-2).$$

This is a joint work with Davila, Dupaigne, K. Wang.

Contact address: wei@math.cuhk.edu.hk

4. LIST OF PARTICIPANTS

Name	Institution
Erin Babinsky	Shenzhen University
Xavier Cabré	ICREA and Universitat Politècnica de Catalunya
Daniele Castorina	University of Roma Tor Vergata
Isabel Coelho	
Benjamin Contri	Université d'Aix-Marseille 3
Serena Dipierro	Universidad de Chile
Alberto Farina	Université de Picardie Jules Verne, Amiens, France
Filomena Feo	Università degli Studi di Napoli
Alessio Figalli	The University of Texas, Austin, USA
Azahara de la Torre	Universitat Politècnica de Catalunya
Manuel del Pino	Universidad de Chile, Santiago, Chile
François Hamel	IUF, Marseille, France
Nikola Kamburov	The University of Arizona, Tucson, USA
Moritz Kassmann	Universität Bielefeld, Germany
Enno Lenzmann	University of Copenhagen, Denmark
Chang-Shou Lin	National Taiwan University, Taipei, Taiwan
Robert McCann	University of Toronto Bahen Centre, Toronto, Canada
Carlo Mantegazza	Scuola Normale Superiore di Pisa, Italy
Luciano Mari	Universidade Federal do Ceará
Alexandre Montaru	Université de Paris 13
Frank Morgan	Williams College, Massachusetts, USA
Nikolai Nadirashvili	CNRS et Université d'Aix-Marseille, France
Miguel Angel Navarro	Universidad de Granada
Manon Nys	Université Libre de Bruxelles
Frank Pacard	É. Polytechnique and U. Paris Est-Créteil, Paris, France
Ireneo Peral	Universidad Autónoma de Madrid, Spain
Joaquin Pérez	Universidad de Granada, Spain
Aldo Pratelli	Universität Erlangen-Nürnberg, Germany
Jean-Michel Roquejoffre	Université Paul Sabatier, Toulouse, France
Matteo Rizzi	
Antonio Ros	Universidad de Granada, Spain
Xavier Ros-Oton	Universitat Politècnica de Catalunya
Manel Sanchon	Universitat de Barcelona
Bernardino Sciunzi	Università della Calabria, Cosenza, Italy
Armin Schikorra	Max Planck Institute für Mathematik
Joaquim Serra Montoli	Universitat Politècnica de Catalunya

Name	Institution
Nicola Soave	Università degli studi di Milano
Joan Solà-Morales Rubió	Universitat Politècnica de Catalunya
Diana Stan	Universidad Autónoma de Madrid
Gabriella Tarantello	Università Tor Vergata, Rome, Italy
Enrico Valdinoci	Università di Milano and Weierstrass Institute, Berlin
Juan Luis Vázquez	Universidad Autónoma de Madrid, Spain
Jun-Cheng Wei	Chinese University of Hong Kong, Shatin, China
Miguel Yangari Sosa	