Workshop
Qdays in Barcelona

CRM, Bellaterra

October 16 to 18, 2013

Acknowledgements: The workshop is partially supported by the CRM, the Generalitat de Catalunya, the European Science Foundation via the network CAST (contact and symplectic Topology), the Clay Mathematics Institute and the Viktor Ginzburg Laboratory (via the Severo Ochoa Program). We would like to express our gratitude to the institutions above for providing financial support that made this program possible.

Organizing and Scientific Committee

Chiara Esposito

Eva Miranda

Francisco Presas

Romero Solha

Speakers

Pierre Bieliavsky (Louvain)

Sergei Gukov (Caltech)

Simone Gutt (Bruxelles-Metz)

Mark Hamilton (Mount Allison)

Peter Hochs (Adelaide)

Stéphane Korvers (Louvain)

Ryszard Nest (Copenhagen)

Paul-Émile Paradan (Montpellier)

Martin Schlichenmaier (Luxembourg)

Jedrzej Sniatycki (Calgary)

Jonathan Weitsman (Northeastern)

Contents

1. Practical Information

Activity documents: Attendance certificates in pdf format will be sent if requested by email.

Computer facilities: The computer space of the CRM will be available for the participants of the conference.

The CRM premises as well as most of the UAB campus have wireless access.

Wifi password: crmwifikey

Library: The library of the Science Building of the UAB will be open from 8:30 am to 7:30 pm on working days.

Breaks: Coffee and cookies will be served during the morning and afternoon breaks to all participants.

Social events: WA welcome cocktail will be organized on Wednesday afternoon
Picture: A group picture will be taken on Wednesday, October 16th before the coffee break. We will inform you of the place to meet. The picture will be posted on the activity's webpage.

Questionnaire: Following the directions of the CRM Governing Board, we give a questionnaire to all the people participating in activities at the CRM in order to assess their level of satisfaction. The questionnaire is anonymous and not mandatory, but we would greatly appreciate it if you could answer the questions. Thank you for your cooperation

Local emergency numbers: General emergency (police, ambulance, firefighters) call 112.

Safety in Barcelona: Although Barcelona is a safe city, please be aware that there is a problem with pickpockets, especially around tourist areas: La Rambla, Plaça Catalunya, Barcelona Airport, major metro and train stations, famous buildings, etc. Be sure to keep your belongings with you at all times, be alert, and be wary of unusual situations.

2. Timetable

Wednesday, October 16	
09:30 – 10:00	Registration
10:00 – 11:00	Jonathan Weitsman (opening) *The Morse index theorem for Hamiltonian loop group spaces and Poincare series of moduli spaces of semistable vector bundles.*
11:00 – 11:30	Coffee break
11:30 – 12:30	Paul-Émile Paradan *Quantization commutes with reduction in the non-compact setting. The case of holomorphic discrete series.*
12:30 – 15:00	Lunch
15:00 – 16:00	Jędrzej Śniatycki *Heisenberg's quantization of co-adjoint orbits of $SO(3)$ and $SL(2,\mathbb{R})$.*
16:00 – 16:30	
16:30 – 17:30	Stéphane Korvers *The deformation quantizations of the Hermitian symmetric space $SU(1,n)/U(n)$.*
17:30	Q-cocktail
Thursday, October 17	
10:00 – 11:00	Pierre Bieliavsky *Strict deformation quantization of compact higher-genus surfaces.*
11:00 – 11:30	Coffee break
11:30 – 12:30	Mark Hamilton *Deformation, convergence, and and invariance of polarization.*
12:30 – 15:00	Lunch
15:00 – 16:00	Martin Schlichenmaier *Some naturally defined star products for Kaehler manifolds.*
16:00 – 16:30	Coffee break
16:30 – 17:30	Peter Hochs *Quantizing tame actions.*

Friday, October 18	
10:00 – 11:00	Simone Gutt *Involutions and representations for reduced quantum algebras.*
11:00 – 11:30	Coffee break
11:30 – 12:30	Sergei Gukov *Quantization via miiro symmetry.*
12:30 – 15:00	Lunch
15:00 – 16:00	Ryszard Nest *Formality for gerbs and quantization.*
16:00 – 16:30	Q-closing with Q-cookies

3. Abstracts of the Speakers

Pierre Bieliavsky
Strict deformation quantization of compact higher-genus surfaces.

Abstract: Similarly as for the noncommutative torus, any compact higher-genus surface can be deformation quantized within a non-formal C^*-algebraic framework in such a way that its space of smooth functions consists in a sub-algebra, and this at every real value of the deformation parameter. The method used to obtain such "smooth" noncommutative surfaces is entirely explicit and applies much more generally. In all its essential aspects, it radically differs from the geometric quantization/Berezin-Toeplitz type methods which other earlier constructions of ("non-smooth") NC surfaces in higher genera are based on.

Contact address: `pierre.bieliavsky@uclouvain.be`

Sergei Gukov
Quantization via mirror symmetry.

Abstract:

Contact address: `gukov@theory.caltech.edu`

Simone Gutt
Involutions and representations for reduced quantum algebras.

Abstract: In the context of deformation quantization, there exist various procedures to deal with the quantization of a reduced space M_{red}. We shall be concerned here with the classical Marsden-Weinstein reduction, assuming that we have a proper action of a Lie group G on a Poisson manifold M, with a moment map J for which zero is a regular value. For the quantization, we follow Bordemann, Herbig and Waldmann (2000), with a simplified approach, and build a star product $\star_{red}$ on M_{red} from a strongly invariant star product $\star$ on M. We address the existence of natural *-involutions on the reduced quantum algebra and the representation theory for such a reduced *-algebra.

We assume that $\star$ is Hermitian and we show that the choice of a formal series of smooth densities on the embedded coisotropic submanifold $C = J^{-1}(0)$, with some equivariance property, defines a *-involution for $\star_{red}$ on the reduced space. Looking into the question whether the corresponding *-involution is the complex conjugation (which is a *-involution in the Marsden-Weinstein context) yields a new notion of quantized unimodular class.

We define a strong Morita equivalence bimodule between $C^\infty(M_{red})[[\lambda]]$ and the finite rank operators on a submodule of $C^\infty(C)[[\lambda]]$.

This is joint work with Stefan Waldmann.

Contact address: `sgutt@ulb.ac.be`

Mark Hamilton
Deformation, convergence, and and invariance of polarization.

Abstract: In the theory of geometric quantization, in order to construct the quantization of a manifold M we need to choose a polarization. The most common examples are a Kaehler polarization, given by a complex structure on M, and a real polarization, given by a foliation of M into Lagrangian submanifolds. If M possesses more than one type of polarization, a natural question is how the resulting quantization depends on the polarization.

One approach to this question, developed by Mourão, Nunes, and collaborators, is to deform the complex structure on M in such a way that the real quantization is a "limit" of the Kaehler quantizations. They have studied this approach for abelian varieties and toric manifolds.

By using a toric degeneration to "approximate" a space by a toric manifold, similar results have been obtained for other spaces as well. I will discuss results for flag varieties (joint with H. Konno), and partial results for weight varieties. In addition, recent work of Harada-Kaveh constructs toric degenerations for a broad class of smooth varieties. We conjecture that the same technique will apply to these varieties as well, which suggests that this type of invariance of polarization holds in considerable generality.

Contact address: `uumbra@gmail.com`

Peter Hochs
Quantizing tame actions.

Abstract: This talk is about a quantization commutes with reduction result that is valid for noncompact groups and noncompact symplectic manifolds, without assuming the orbit space to be compact or the momentum map to be proper. It is assumed that the symplectic reduction at zero is compact, and that the action is tame. Tameness is a geometric condition on the action, which implies a growth condition on the square of the Dirac operator. It is satisfied in various classes of examples, such as cocompact actions, actions by subgroups on strongly elliptic coadjoint orbits, and actions by Lie groups on their cotangent bundles.

Although this result is not stated in terms of K-theory and operator algebras, it leads to a version of quantization with values in the K-homology of the reduced C^*-algebra of the group acting. This is a simultaneous generalization of the generalized representation ring of a compact group, and the K-theory of the C^*-algebra of any Lie group. Under certain conditions, this K-homological quantization reduces to the definition used by Ma-Zhang and Paradan for compact groups, and the one used by Landsman for compact orbit spaces.

This is joint work with Mathai Varghese.

Contact address: `peter.hochs@adelaide.edu.au`

Stéphane Korvers

The deformation quantizations of the Hermitian symmetric space $SU(1,n)/U(n)$.

Abstract: In "The Deformation Quantizations of the Hyperbolic Plane" (P. Bieliavsky, S. Detournay, P. Spindel, Commun. Math. Phys. 2008), the authors show that a curvature contraction on the hyperbolic plane produces a symplectic symmetric surface whose transvection group is isomorphic to the Poincaré group in dimension 2. They also prove that from this contraction process emerges a differential operator of order two whose certain solutions of its evolution equation define convolution operators that intertwine the deformation theory (star-products) at the contracted level with that of the hyperbolic plane. This talk will be devoted to the study of a generalization of this construction in the case of the Hermitian symmetric space $SU(1,n)/U(n)$.

Contact address: `stephane.korvers@uclouvain.be`

Ryszard Nest

Formality for gerbs and quantization.

Abstract: We will formulate the formality theorem for manifolds with a gerbe (both smooth and analytic case), sketch a proof and applications to the classification of quantization of manifolds with gerbes.

Contact address: `rnest@math.ku.dk`

Paul-Émile Paradan

Quantization commutes with reduction in the non-compact setting. The case of holomorphic discrete series.

Abstract: Let V be an irreducible representation of highest weight λ of a compact Lie group G. In the early 80s, Gert Heckman and Guillemin-Sternberg showed that the multiplicities of V with respect to a subgroup H depends of the Hamiltonian action of H on the coadjoint orbit $G\lambda$. This result had given rise to the famous conjecture of Guillemin-Sternberg "*Quantization commutes with reduction*, denoted $[Q,R]=0$. Conjecture finally proved in full generality by Meinrenken-Sjamaar in the late 90s.

In this talk, I will explain why this property is still valid when V is a representation of the holomorphic discrete series of a real reductive group G, and H is a reductive subgroup. The novelty of this result is that we obtain a theorem of the type $[Q,R]=0$ with a group of symmetry H that is non-compact. The main tool for proving this result is the "formal geometric quantization" procedure developed by Ma-Zhang and myself.

Contact address: `paradan@math.univ-montp2.fr`

Martin Schlichenmaier

Some naturally defined star products for Kaehler manifolds.

Abstract: We give for the Kähler manifold case an overview of the constructions of some naturally defined star products. In particular, the Berezin-Toeplitz, Berezin, Geometric Quantization, Bordemann-Waldmann, and Karbegov standard star product are introduced. With the exception of the Geometric Quantization case they are of separation of variables type. The classifying Karabegov forms and the Deligne-Fedosov classes are given. Besides the Bordemann-Waldmann star product they are all equivalent.

Contact address: `Martin.Schlichenmaier@uni.lu`

Jędrzej Śniatycki

Heisenberg's quantization of co-adjoint orbits of $SO(3)$ ***and*** $SL(2,\mathbb{R})$***.***

Abstract: A completely integrable Hamiltonian system, admits local action-angle variables. If there is no monodromy, actions can be globally defined, and the Bohr-Sommerfeld conditions give the joint spectrum of the actions that has the structure of a lattice. Assuming that all eigenvalues of the actions are simple, we can construct a Hilbert space $\mathcal{H}$ spanned by eigenvectors of the actions.

The lattice structure of the basis in $\mathcal{H}$ consisting of eigenvectors of the actions gives rise to shifting operators in $\mathcal{H}$. For each generator of the lattice, the operator of shifting in the direction of this generator may be interpreted as quantization of the the exponential of the corresponding angle multiplied by i. This interpretation gives rise to a quantization of the Poisson algebra consisting of smooth functions of actions multiplied by linear combinations of sines and cosines of the angles. We refer to the resulting quantum theory as the Bohr-Sommerfeld-Heisenberg quantization or B.S.H. quantization.

In the lecture, I shall present the B.S.H. quantization of coadjoint orbits of $SO(3)$ and $SU(2)$. I shall also discuss an attempt to generalize the B.S.H. quantization to completely integrable systems with monodromy.

Contact address: `sniatyck@ucalgary.ca`

Jonathan Weitsman

The Morse index theorem for Hamiltonian loop group spaces and Poincare series of moduli spaces of semistable vector bundles.

Abstract: In previous work (*Invent. Math.* **155**, 225-251 (2004)) we used a version of Morse theory to prove an analog of the Kirwan surjectivity theorem for Hamiltonian loop group spaces. In this paper we continue the study of Morse theory for these spaces by giving an effective formula for the index of our Morse

function at its critical points. We apply this formula to the computation of the Poincaré series of moduli spaces of semistable vector bundles.
This is joint work with R. Bott and S. Tolman.

Contact address: j.weitsman@neu.edu

4. List of Participants

Name	Institution
Romero Barbieri Solha	Universitat Politècnica de Catalunya
Andratx Bellmunt Giralt	Universitat de Barcelona
Pierre Bieliavsky	Université catholique de Louvain
Michel Cahen	Université Libre de Bruxelles
Hector Castejon Diaz	University of Luxembourg
Nicola Ciccoli	
Alvaro del Pino Gómez	Universidad Autónoma de Madrid
Chiara Esposito	Mathematisches Forschungsinstitut Oberwolfach
Sergei Gukov	California Institut of Technology
Simone Gutt	Université Libre de Bruxelles
Mark Hamilton	Mount Allison University
Jérémy Haut	Université Catholique de Louvain
Peter Hochs	The University of Adelaide
Cheikh Khoule	Université Cheikh Anta Diop
Anna Kiesenhofer	Universitat Politècnica de Catalunya
Stéphane Korvers	Université catholique de Louvain
Runliang Lin	National Tsing Hua University
Eva Miranda	Universitat Politècnica de Catalunya
Hassan Najafi Alishah	Universidade de Lisboa
Ryszard Nest	University of Copenhagen
Mohammed Olanrewaju	University of Ilorin
Paul-Émile Paradan	Université Montpellier
Francisco Presas	ICMAT
Abdó Roig-Maranges	Universitat Politècnica de Catalunya
Vladimir Salnikov	INSA de Rouen
Nicola Sansonetto	Padova University
Yasha Savelyev	ICMAT
Martin Schlichenmaier	University of Luxembourg
Jedrzej Sniatycki	University of Calgary
Alex Thiele	Universitat Politècnica de Catalunya
Andreas Vollmer	Friedrich-Schiller Universität Jena
Jonathan Weitsman	Northeastern University