THE CRM APPLIED MATHEMATICAL PHYSICS (CAMP) SEMINARS

CENTRE DE RECERCA MATEMÀTICA

Daniele Avitabile

University of Nottingham

Analysing coherent structures via interfacial dynamics: from spatio-temporal canards to coarse-grained computations.

Abstract:

I will discuss level-set based approaches to study the existence and bifurcation structure of spatio-temporal patterns in biological neural networks. Using this framework, which extends previous ideas in the study of neural field models, we study the first example of canards in an infinite-dimensional dynamical system, and perform a computational reduction of dimensionality in certain neural network models.

Phenomenological neural field models have been intensively studied in the past and are known to support a variety of coherent structures observed experimentally (localised bumps of activity, travelling fronts, travelling bumps, lurching waves, rotating waves). These models are typically written as integro-differential equations, where the integral term is a Hammerstein nonlinear operator, featuring a sigmoidal firing rate and a synaptic kernel. Successful strategies for the analysis of these models include special choices of the synaptic kernels (leading to equivalent PDE formulations) and interface methods.

The main message of the talk is that the latter can be used effectively to construct or compute coherent structures in multiple-scale, heterogeneous, and possibly stochastic systems. (***Continued on the next page***)

Date:	**May 4, 2017**
Place:	**Room C1/028**
Time:	**12:00**

Abstract (continuation):

I will initially consider a spatially-extended network with heterogeneous synaptic kernel. Interfacial methods allow for the explicit construction of a bifurcation equation for localised steady states, so that analytical, closed-form expressions for a classical "snakes and ladders" bifurcation scenario can be derived.

When the model is subject to slow variations in the control parameters, a new type of coherent structure emerges: the structure displays a spatially-localised pattern, undergoing a slow-fast modulation at the core. Using interfacial dynamics and geometric singular perturbation theory, we show that these patterns follow an invariant repelling slow manifold, hence we name them "spatio-temporal canards". We classify spatio-temporal canards and give conditions for the existence of folded-saddle and folded-node canards. We also find that these structures are robust to changes in the synaptic connectivity and firing rate. The theory correctly predicts the existence of spatio-temporal canards with octahedral symmetries in a neural field model posed on a spherical domain.

I will then discuss how the insight gained with interfacial dynamics may be used to perform coarse-grained bifurcation analysis on neural networks, even in models where the network does not evolve according to an integro-differential equation. Time permitting, two illustrative examples will be discussed.

The first example is a well-known event-driven network of spiking neurons, proposed by Laing and Chow. In this setting, we construct numerically travelling waves whose profiles possess an arbitrary number of spikes. An open question is the origin of the travelling waves, which have been conjectured to form via a destabilisation of a bump solution. We provide numerical evidence that this mechanism is not in place, by showing that disconnected branches of travelling waves with countably many spikes exist, and terminate at grazing points; the grazing points correspond to travelling waves with an increasing number of spikes, a well-defined width, and decreasing propagation speed.

The second example is a heterogeneous neural network written as a discrete Markov chain with discrete ternary state space, posed on a lattice. The model supports coarse bumps, multi-bumps and travelling waves, but the derivation of a coarse evolution equation is nontrivial. I will show that, by choosing the interfaces as coarse variables, it is possible to perform an efficient numerical coarse-graining, following the pattern in parameter space and analysing their stability.