Title: Arnold diffusion in the elliptic restricted three-body problem

Authors: Maciej Capiński, Marian Gidea, Rafael de la Llave

Abstract:

We consider the planar elliptic restricted three-body problem as a perturbation of the planar circular restricted three-body problem, with the eccentricity of the binary regarded as the perturbation parameter. We show that there exist trajectories whose energy changes between two given levels, for all sufficiently small eccentricities. Equivalently, there exist trajectories that start near the Lyapunov orbit of the unperturbed problem for some given energy, and end up near the Lyapunov orbit of the unperturbed problem for some other given energy.

The proof combines analytical arguments with numerical verifications. We show the existence of a normally hyperbolic invariant manifold, and we study the 'inner dynamics', given by the restriction of the flow to the manifold, and the 'outer dynamics' given by the heteroclinic connections to the manifold. On the inner dynamics, we do not need to verify any special conditions, and implicitly we do not use the KAM theorem (or Aubry-Mather theory); we only use the Poincaré recurrence theorem. On the outer dynamics, we identify a family of scattering maps that satisfy certain non-degeneracy conditions, which we verify via Melnikov type computations. We produce pseudo-orbits, given by alternately applying the inner dynamics and the outer dynamics, along which the energy increases in a prescribed way. Then we apply some topological shadowing lemma, which allows one to find true orbits near that pseudo-orbit.

The topological shadowing lemma referred above can be found in [1]. It can be used in both analytical arguments and in rigorous numerical experiments. In particular, it can be applied to establish the existence of diffusing orbits in a priori unstable Hamiltonian systems of arbitrary dimension.

References:

[1] Marian Gidea, Rafael de la Llave, Tere Seara, A General Mechanism of Diffusion in Hamiltonian Systems: Qualitative Results, arXiv:1405.0866