Abstracts of the Speakers

Richard Garner
Combinatorial structure of type dependency.

Abstract: For a given dependent type theory T, the collection of type theories which extend T by the addition of new types, terms and equations form a locally finitely presentable category. This category is therefore equivalent to the category of algebras for a well-behaved monad on a presheaf category. The question then arises: which presheaf category, and which monad? This talk will document some of my attempts to give a sensible answer to this question.

Contact address: `richard.garner@mq.edu.au`

André Joyal
A categorical approach to homotopy type theory.

Abstract: We give a categorical axiomatisation of homotopy type theory based on the Gambino-Garner factorisation system. If times permits, we will construct models from higher toposes.

Contact address: `joyal.andre@uqam.ca`

Peter LeFanu Lumsdaine
The Blakers-Massey theorem in homotopy type theory.

Abstract: One of the main themes to emerge from the 2013 Special Year in Univalent Foundations was a sequence of theorems recreating results of classical homotopy theory within type theory. Some proofs were novel, while others were more closely based on classical approaches.

One such result, the Blakers-Massey connectivity theorem, is of particular interest in that all classical proofs use some specifics of their settings (usually, Top or SSet) not available to us; so the type-theoretic proof was, by necessity, new in parts. This allowed us, as a by-product, to translate the proof back into classical language and obtain the theorem in wider generality than was previously known: we show that it holds in any o-topos (in the sense of Lurie).

I will introduce the Blakers-Massey theorem and our approach to it, and discuss the process of translating a type-theoretic proof into o-categorical language. This material was joint work with (of couse) many participants in the special year, but especially with Eric Finster and Dan Licata.

Contact address:`p.l.lumsdaine@mathstat.dal.ca`

Thomas Streicher

How intensional is homotopy type theory?

Abstract: I will recall some old work of mine where I formulated criteria for intensionality for dependent type theories ‘a la Martin-Loef and constructed models for them. I will give a simplified such model in my talk.

Homotopy type theory does not validate these criteria since the Univalence Axiom entails the principle of Function Extensionality from which it would follow that the set of provable $\Pi 0_1$ sentences were decidable which, however, is not the case for any r.e. consistent extension of PRA (Primitive Recursive Arithmetic).

Moreover, I will sketch a proof that Homotopy Type Theory is conservative over intensional type theory extended by function extensionality w.r.t. propositions which can be formulated without reference to a universe.

Contact address: `streicher@mathematik.tu-darmstadt.de`

Abstracts of the Contributed Talks

Steve Awodey
Survey of Univalent Foundations.

Abstract: I will give a survey of Univalent Foundations.

Contact address:

Bruno Barras
Native implementation of a subclass of higher-inductive types in Coq: progress report.

Abstract: Higher Inductive Types (HITs) form a convenient way (inspired by indutive types) to define homotopic types with a better control on the loop spaces. However, there is currently no proof assistant that support HITs natively. Both Agda and Coq use a hack (the so-called private inductive types) to model them, but they have serious limitations we will briefly enumerate. The main topic of this talk is to make a progress report of our first attempt to implement HITs. We will discuss the design choices and the open problems. More specifically, we will investigate the possibility to have pattern-matching and structural fixpoint as independent primitives, as a refinement of the usual primitive recursor.

Contact address: `bruno.barras@inria.fr`

Guillaume Brunerie
The Hopf fibration.

Abstract: I will present a construction of the Hopf fibration in homotopy type theory and a proof that its total space is the 3-sphere. I will then explain what issues arise when trying to formalize the proof in Coq or Agda and how they could be solved by adding more definitional equalities to the type theory.

Contact address:

Favonia
Covering spaces in homotopy type theory.

Abstract: Covering spaces play an important role in classical homotopy theory, whose algebraic characteristics have deep connections with fundamental groups of underlying spaces. It is natural to ask whether these connections can be stated in homotopy type theory, that is, phrased in a completely homotopy-invariant manner. This talk will summarize my work in progress, which is to recover the classical results (e.g. the classification theorem) so as to explore the expressiveness

of these new foundations. Some interesting techniques employed in the current proofs seem applicable to other constructions as well.

Contact address: favonia@cmu.edu

Dan Grayson
Proof assistant design.

Abstract: Last February Voevodsky introduced the idea of a "homotopy type system" with two types of identity types and two sorts of types: fibrant types and general types. I will discuss various ideas for implementing a proof assistant based on it, with an eye toward understanding the semantic model in simplicial sets as choices in the design of the type system itself are made, as well as dealing with a notion of definitional equality subject to proof, and hence not decidable.

Contact address: dan@math.uiuc.edu

Hugo Herbelin
Constructing simplicial sets.

Abstract: We investigate how to define simplicial sets in such a way that the property of being degenerate is decidable.

In a first step, we consider a definition where degeneracies are considered algebraically but faces are axiomatized.

In a second step, we extend Lumsdaine and Awodey's proposal for inductively constructing dependently-typed semi-simplicial sets into a construction of simplicial sets. In this case, both degeneracies and faces are hard-wired in the structure.

We finally discuss some issues in defining simplicial types.

Contact address:

Simon Huber
A model of type theory in cubical sets.

Abstract: We present a model of intensional Martin-Löf Type Theory with function extensionality expressed in a constructive meta-theory. The model is based on a variant of cubical sets and can be seen as a constructive version of Voevodsky's model based on Kan simplicial sets.

This is a joint work with Marc Bezem and Thierry Coquand.

Contact address: simonhu@chalmers.se

Krzysztof Kapulkin

Univalent categories and the Rezk completion.

Abstract: When formalizing category theory in traditional, set-theoretic foundations, a significant discrepancy between the foundational notion of "sameness" –equality– and its categorical notion arises: most category-theoretic concepts are invariant under weaker notions of sameness than equality, namely isomorphism in a category or equivalence of categories. We show that this discrepancy can be avoided when formalizing category theory in Univalent Foundations.

The *Univalent Foundations* is an extension of Martin-Löf Type Theory (MLTT) recently proposed by V. Voevodsky [4]. Its novelty is the *Univalence Axiom* (UA) which closes an unfortunate incompleteness of MLTT by providing "more equalities between types". This is obtained by identifying equality of types with equivalence of types. To prove two types equal, it thus suffices to construct an equivalence between them.

When formalizing category theory in the Univalent Foundations, the idea of Univalence carries over. We define a *precategory* to be given by a type of objects and, for each pair (x, y) of objects, a *set* $\hom(x, y)$ of morphisms, together with identity and composition operations, subject to the usual axioms. In the Univalent Foundations, a type X is called a *set* if it satisfies the principle of Uniqueness of Identity Proofs, that is, for any $x, y \colon X$ and $p, q \colon \mathrm{Id}(x, y)$, the type $\mathrm{Id}(p, q)$ is inhabited. This requirement avoids the introduction of coherence axioms for associativity and unitality of categories.

A *univalent* category is then defined to be a category where the type of isomorphisms between any pair of objects is equivalent to the identity type between them. We develop the basic theory of such univalent categories: functors, natural transformations, adjunctions, equivalences, and the Yoneda lemma.

Two categories are called *equivalent* if there is a pair of adjoint functors between them for which the unit and counit are natural isomorphisms. Given two categories, one may ask whether they are equal in the type-theoretic sense –that is, if there is an identity term between them in the type of categories– or whether they are equivalent. One of our main results states that for univalent categories, the notion of (type-theoretic) *equality* and (category-theoretic) *equivalence coincide*. This implies that properties of univalent categories are automatically invariant under equivalence of categories –an important difference to the classical notion of categories in set theory, where this invariance does not hold.

Moreover, we show that any category is weakly equivalent to a univalent category –its *Rezk completion*– in a universal way. It can be considered as a truncated version of the Rezk completion for Segal spaces [3]. The Rezk completion of a category is constructed via the Yoneda embedding of a category into its presheaf category, a construction analogous to the *strictification* of bicategories by the Yoneda embedding into Cat, the 2-category of categories.

Large parts of this development have been formally verified [1] in the proof assistant **Coq**, building on Voevodsky's *Foundations* library [5]. In particular, the formalization includes the Rezk completion together with its universal property.

A preprint covering the content of this talk is available on the arXiv [2].

This is a joint work with Benedikt Ahrens and Michael Shulman.

Acknowledgments. The authors thank Vladimir Voevodsky for many helpful conversations.

This material is based upon work supported by the National Science Foundation. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation

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[5] Vladimir Voevodsky. *Univalent foundations repository.* ongoing **Coq** development.

Contact address: `krk56@pitt.edu`

Yves Lafont

Towards a simple definition of weak infinity-categories.

Abstract: I seek for the holly grail: a simple and natural definition of weak infinity-categories and/or groupoids. Up to know, I only work in the framework of strict infinity-categories [1, 2], even if we considered weak inverses in [2].

It seems that the existing definitions of weak infinity-groupoids are rather complicated and/or indirect. In particular, Homotopy Type Theory yields a natural definition, but it cannot be considered as an algebraic definition, since it uses a logical system whose syntax is rather sophisticated. Indeed, I believe that such a system is very useful for the foundations of mathematics and for the design of proof assistants, but it should not be considered as a prerequisite for algebraic definitions. Hence, I would like to extract a direct definition from this indirect one.

Furthermore, I would like to use rewriting to deduce the coherence conditions: the idea is to generalize Mac Lanes's argument to higher dimension. This is related to another problem: find a simple and natural definition of Street orientals.

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Contact address: `yves.lafont@univ-amu.fr`

Zhaohui Luo

H-propositions in an impredicative type theory.

Abstract: In this talk, we shall study the notion of h-propositions, also called mere propositions [Pro13], in the type theory uTT where there is an impredicative universe *Prop* of logical propositions. In particular, we show that, in uTT, a type is an h-proposition if and only if it is equivalent (and equal, by univalence) to a proposition of type *Prop*.

uTT is the extension of the type theory UTT [Luo94]. UTT combines Martin-Löf's intensional type theory with the Calculus of Constructions, with its predicative universes $Type_i$ containing 'data types' and the impredicative universe *Prop* the logical propositions. It was implemented in the proof assistants Lego [LP92] and Plastic [CL01] and is very much similar to the current type theory implemented in Coq [Coq07], especially after the universe *Set* in Coq became predicative. UTT's meta-theoretic properties such as strong normalisation were studied by Goguen in his PhD thesis [Gog94].

uTT extends UTT with the following:

- The univalence axioms $UA(U)$, as proposed by Voevodsky, for all predicative universes $U \in \{Type_i \mid i \in \omega\}$, assuming that the canonical map from the identity type $Id(U, A, B)$ to the type of equivalences between A and B be an equivalence.
- The inference rule of proof irrelevance for propositions in the impredicative universe *Prop*:

$$\frac{\Gamma \vdash P : \ Prop \quad \Gamma \vdash p : P \quad \Gamma \vdash q : P}{\Gamma \vdash p = q : P}. \tag{PI}$$

- An equality proposition $Eq(A, a, b) : Prop$ with large elimination[1], for any type A and objects $a, b : A$.

Note that, except univalence, the other two extensions are very natural and similar ideas have been studied in different contexts from HoTT: for example, proof irrelevance in [Coq90, Wer08] and $Eq(A, a, b)$ in §9.3 of [Luo94]. However,

[1] The Eq-elimination is large in that it eliminates over all type-indexed families $C : (x, y : A)(Eq(A, x, y))Type$, where $Type$ is the kind of all types in the logical framework. It is worth remarking that $Eq(A, a, b)$ is logically equivalent to $Id(A, a, b)$, although proof irrelevance holds for the former but not for the latter.

they have interesting implications for the theory of HoTT, including the following theorem.

Theorem. *Let* $U \in \{Type_i \,|\, i \in \omega\}$ *be a predicative universe (note that, informally,* $Prop \subseteq U$.[2]*) Then,* $A\colon U$ *is an h-proposition if and only if* $Eq(U, A, P)$ *for some* $P\colon Prop$. *In symbols, defining*

$$isHProp(A) =_{df} \quad \forall x, y\colon A.\ Eq(A, x, y)$$
$$isProp(A) =_{df} \exists P\colon Prop.\ Eq(U, A, P)$$

we have $isHProp(A) \Leftrightarrow isProp(A)$.

Proof sketch. The sufficiency is straightforward by (PI).[3] For necessity, it suffices to show that, for any h-proposition $A : U, Eq(U, \mathbf{1}, A)$ is equivalent (and equal, by univalence) to A. □

Remark. According to the above theorem, one may argue that the type *Prop* in uTT is an adequate totality of propositions that also captures the notion of h-proposition. As a consequence, propositional resizing [Pro13, Voe11] is not necessary in uTT.[4]

In this talk, besides impredicativity and the above theorem, we shall also discuss the logic in uTT and non-propositional resizing [Voe11], among other things.

This is joint work with Fedor Part.

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[2]Formally we have explicit lifting operators from *Prop* to the predicative universes.

[3]Technically, for a proof to go through, it would be enough for one to assume an axiom of proof irrelevance: $UP : \forall P : Prop : isHProp(P)$. However, one may argue that the stronger (PI) is more appropriate; for example, the property of equality reflection may hold with (PI) but not with UP.

[4]This is related to a remark in §3.5 of [Pro13] about a type $\Omega : U$ which 'classifies all mere propositions'. One may regard *Prop* as such a type.

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Contact address: zhaohui.luo@hotmail.co.uk

Jack Morava
From algebra to geometry and back again.

Abstract: Work in the mid-1600's by Fermat, Descartes, and others created our current understanding of algebra and geometry as the same subject, united by a contravariant correspondence, and the 20th century pushed this further by constructing a triangulated monoidal localization of the category of topological spaces. Since then it has become clear that a certain moduli stack (of formal groups) is a natural geometric repository for this algebraization in terms of spectra.

[This is part of the movement from set to category theory as a useful foundation for mathematics, and it involves an enormous amount of (quite technical) work; but I will try to keep that in the background, and focus on broader themes.]

Contact address: jack@math.jhu.edu

Paige North
Towards a model of HoTT in topological spaces.

Abstract: The existence of a model of homotopy type theory in simplicial sets has been motivational in the subject. However, the existence of any other models has been unknown. I will explain the extent to which homotopy type theory can be interpreted in the model structures on topological spaces, and what this tells about these model structures.

Contact address: paigenorth@gmail.com

Erik Palmgren
Constructing categories and setoids of setoids in type theory.

Abstract: In this talk we consider the problem of building rich categories of setoids in standard intensional Martin-Löf's type theory (MLTT), and in particular how to handle the problem of equality on objects in this context. We show that any (proof-irrelevant) family of setoids over a setoid gives rise to a category with object equality. Such a family may be obtained from Aczel's model construction of CZF in type theory. It is proved that the category obtained is isomorphic to the internal category of sets in this model. We also show that Aczel's model construction may be extended to include the elements of any setoid as atoms or urelements. We moreover obtain a natural extension of CZF, adding atoms. This

extension, CZFU, is validated by the extended model. The main theorems have been checked in the proof assistant Coq which is based on MLTT. We also give an alternative description of the category of setoids inside MLTT.

This is partly joint work with Olov Wilander.

I could present these papers in a talk
`http://people.su.se/~epalm/Yet_another_category_of_setoids.pdf`
`http://people.su.se/~epalm/coq/czf_and_setoids/czf&setoids.pdf`

Contact address:

Emily Riehl
Made-to-order weak factorization systems.

Abstract: Quillen, who popularized weak factorization systems in abstract homotopy theory, also introduced a general procedure for constructing functorial factorizations with desired lifting properties. This talk will survey recent extensions of this "small object argument" which can be used to expand its applicability. The first two of these - the"algebraic perspective" and the use of enrichment - extend the traditional notion of a cofibrantly generated weak factorization system. We'll also discuss techniques that can be used even in the non cofibrantly generated case and describe the particularly close connection between the functorial factorizations so-produced and the lifting properties these functors are designed to satisfy.

Contact address: `eriehl@math.harvard.edu`

Egbert Rijke
A descent theorem in the univalent foundations.

Abstract: A univalent universe acts as an object classifier and therefore we can identify functions to a type with dependent types over that type. This allows for rewriting the descent condition of higher toposes to a condition formulated dependently. Thus, equifibered (or cartesian) functors between diagrams become dependent diagrams with an equifiberedness condition. A proof of the descent condition, that the class of equifibered diagrams is equivalent to the class of functions into the homotopy colimit of the base diagram, comes out very naturally in this setting. In the present work, we have only considered diagrams over graphs (covering at least all the colimits of the recently published HoTT book).

Contact address: `e.m.rijke@gmail.com`

Urs Schreiber
Synthetic quantum field theory.

Abstract: Attempts to axiomatize classical continuum physics inside toposes famously had let W. Lawvere to his "synthetic" axioms for differential geometry formulated internal to toposes, and more recently to his formulation of "axiomatic cohesio". In this talk I report on interesting results that one finds when interpreting these axioms not in ordinary toposes, but in their homotopy theoretic incarnation as infinity-toposes; in other words, when implementing them in homotopy type theory. The central claim is that this way one obtains beyond a synthetic theory of differential equations and hence of classical physics a natural internal formulation of modern fundamental physics, namely of local gauge quantum field theory. I will try to explain this in general and go through some examples.

This formalization of "axiomatic cohesion" in homotopy type theory is joint work with Mike Shulman, see here: `http://ncatlab.org/schreiber/show/Quantum+gauge+field+theory+in+Cohesive+homotopy+type+theory`.

My talk will follow a subset of the pdf slides here: `http://ncatlab.org/schreiber/show/Synthetic+Quantum+Field+Theory` which I had previously shown at the CMS summer meeting in Halifax.

Contact address: `urs.schreiber@gmail.com`

Kristina Sojakova
Higher inductive types as homotopy-initial algebras.

Abstract: We consider a propositional version of higher inductive types, where the (dependent) computation rules are stated up to homotopy instead of a definitional equality. One advantage of this approach is that such an inductive type X can now be axiomatized using the type theory itself rather than a meta-language; we can for instance form the type of intervals, the type of n-spheres, toruses, etc.

Furthermore, we obtain a compact characterization of such higher inductive types as homotopy-inital algebras. An algebra is a type X together with a number of finitary operations $f, g, h \ldots$, which are allowed to act not only on X but also on any higher identity type over X. An algebra homomorphism has to preserve all operations up to a higher homotopy. Finally, an algebra X is homotopy-initial if the type of homomorphisms from X to any other algebra Y is contractible.

Using these notions, we can show that any interval type gives rise to a homotopy-initial algebra and conversely, any homotopy-inital algebra of a certain form defines an interval type. This correspondence can be shown to give an equivalence of types between the type of intervals and the type of homotopy-initial algebras, thus providing a complete characterization of the interval type.

Contact address: `kristinas@cmu,edu`

Bas Spitters
Sets in homotopy type theory.

Abstract: Homotopy Type Theory with higher inductive types and the univalence axiom is conjectured to be the internal language for oo-toposes.

As such it is proposed as a new foundation for mathematics. We wonder whether the traditional set theoretical foundations can be reconstructed inside this framework. We prove that sets in homotopy type theory form a PiW-pretopos. This is similar to the fact that the 0-truncation of an oo-topos is a topos.

The type theoretical universe of sets has a a subobject classifier as well as a 0-object classifier, itself being a groupoid.

Both of these are large; assuming certain resizing rules we actually obtain a topos. We also discuss the axiom of multiple choice.

[Some of these results can also be found in the book on homotopy type theory. We will make some connections with the formalizations in Coq.]

This is a joint work with Egbert Rijke.

Contact address: