2nd Barcelona Weekend on Operator Algebra and NonCommutative Algebra

Centre de Recerca Matemàtica

September 19th to 21st, 2019

Abstracts Book

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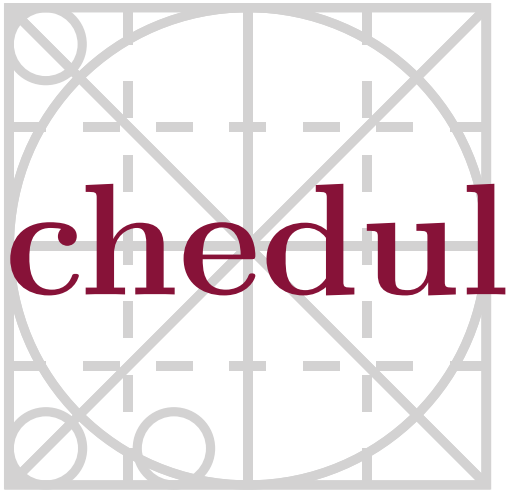


Thursday 19th of September, 2019

14:00–14:30	Registration
14:30–15:30	**Realization problems for monoids of projectives or projections** Ken Goodearl, *University of California at Santa Barbara*
15:30–16:00	Coffee Break
16:00–17:00	**Some computer-assisted ring theory** Kevin C. O'Meara, *University of Canterbury*
17:00–18:00	**A sheaf cohomology theory for C*-algebras** Martin Mathieu, *Queen's University Belfast*

Friday 20th of September, 2019

09:30–10:30	**Diagonal dimension for C^*-pairs** Kang Li, *Polish Academy of Sciences*
10:30–11:00	Coffee Break
11:00-12:00	**Largest ideals in Leavitt path algebras** Mercedes Siles Molina, *Departamento de Álgebra, Geometría y Topología, Universidad de Málaga*
12:00-13:00	**Homotopy classification of Leavitt path algebras** Guillermo Cortiñas, *Universidad de Buenos Aires*
13:00–14:30	Lunch
14:30–15:30	**Classification of Leavitt path algebras via graded K-theory** Roozbeh Hazrat, *Western Sydney University*
15:30–16:00	Coffee Break
16:00–17:00	**Anti-elementarity for ranges of functors** Friedrich Wehrung, *LMNO, CNRS Université de Caen*
17:00–18:00	**Duality for rings constructed from groupoids** Aidan Sims, *University of Wollongong*

Saturday 21st of September, 2019

10:00–11:00	**On the K-theory of ample groupoid C*-algebras** Christian Bönicke, *University of Glasgow*
11:00–11:30	Coffee Break
11:30–12:30	**On the Dichotomy of Paradoxical versus Amenable** Fernando Lledó, *University Carlos III de Madrid and ICMAT*

Abstracts

of the

Plenary Talks

On the K-theory of ample groupoid C*-algebras

Christian Bönicke

University of Glasgow, Glasgow, G12 8QQ, Scotland

E-mail address: christian.bonicke@glasgow.ac.uk

Many interesting C*-algebras can be realized as the (twisted) C*-algebra of an ample groupoid. Hence there is considerable interest in understanding their K-theory. This is a very difficult problem. One approach is given by the Baum-Connes conjecture, which asserts that a certain assembly map

$$\mu_A : \mathrm{K}_*^{\mathrm{top}}(G; A) \to \mathrm{K}_*(A \rtimes_r G)$$

from the topological K-theory of the groupoid G with coefficients in a G-C*-algebra A into the K-theory of the associated reduced crossed product is an isomorphism. In this talk I will present a method that allows one to deal with certain questions concerning the left hand side of the assembly map: The Going-Down principle. I will illustrate how one can use this method to prove results about the conjecture, and in cases where the conjecture is known to hold, derive interesting consequences for the K-theory of (twisted) groupoid C*-algebras.

Homotopy classification of Leavitt path algebras

Guillermo Cortiñas

Universidad de Buenos Aires, Ayacucho 1245 (C1111AAI) - Buenos Aires, Argentina

E-mail address: gcorti@dm.uba.ar

A result of Cuntz and Rørdam ([3]) says that the simple graph algebras associated to finite graphs, i.e. the simple Cuntz-Krieger algebras, are classified up to isomorphism by the pair $(K_0, [1])$ consisting of the Grothendieck group and its order unit. It is an open question whether a similar result holds for Leavitt path algebras [1]. In the talk we will discuss joint work with Diego Montero [2] where we show that $(K_0, [1])$ is a complete invariant for the classification up to homotopy of the simple Leavitt path algebras of finite graphs.

References

[1] Gene Abrams and Adel Louly and Enrique Pardo and Christopher Smith, Flow invariants in the classification of Leavitt path algebras, J. Algebra 333 (2011) 202–231.

[2] Guillermo Cortiñas and Diego Montero, Homotopy classification of Leavitt path algebras. arXiv:1806.09242.

[3] Mikel Rørdam, Classification of Cuntz-Krieger algebras. K-theory 9 (1995), no.1, 31–58.

Realization problems for monoids of projectives or projections

Ken Goodearl

University of California at Santa Barbara, Santa Barbara, California 93106, United States of America

E-mail address: goodearl@math.ucsb.edu

We will discuss the problem of *realization* of a (commutative) monoid M as the monoid $V(R)$ of isomorphism classes of finitely generated projective modules over a ring R from a given class. There are parallel realization problems over C*-algebras R, where $V(R)$ can be presented as the monoid of equivalence classes of projections from matrix algebras over R. In particular, realization problems encompass questions of whether various types of cancellation hold for projectives (resp., projections) over a given class of rings (resp., C*-algebras).

We will survey positive and negative results to date, concentrating on realization of *conical refinement monoids* by von Neumann regular rings and/or C*-algebras with real rank 0. Conicality is the condition ($x + y = 0 \implies x = y = 0$), which holds for any $V(R)$, while refinement is the property ($x_1 + x_2 = y_1 + y_2 \implies x_i = z_{i1} + z_{i2}$ and $y_j = z_{1j} + z_{2j}$ for some z_{ij}), which holds for $V$of any von Neumann regular ring and V of any C*-algebra with real rank 0.

Classification of Leavitt path algebras via graded K-theory

Roozbeh Hazrat

(in collaboration with Pere Ara and Huanhuan Li)

Western Sydney University, Sydney, NSW, Australia

E-mail address: r.hazrat@westernsydney.edu.au

We survey the results obtained to date on the classification of Leavitt path algebras via the graded K-theory. Several parts of the talk is a joint work with Pere Ara and Huanhuan Li.

Diagonal dimension for C^*-pairs

Kang Li

(in collaboration with Hung-Chang Liao and Wilhelm Winter)

Polish Academy of Sciences, Palace of Culture and Science, Plac Defilad 1, 00-901 Warsaw, Poland

E-mail address: kli@impan.pl

We will introduce the notion of diagonal dimension for diagonal pairs of C^*-algebras in the sense of Kumjian, and will compare it with the usual nuclear dimension for C^*-algebras. For instance, the Jiang-Su algebra $\mathcal{Z}$ admits a diagonal MASA D such that the diagonal dimension of $(\mathcal{Z}, D)$ is equal to n for any given natural number n even though the nuclear dimension of $\mathcal{Z}$ is equal to 1. We also show that the diagonal dimension of a uniform Roe algebra with respect to the standard diagonal is equal to the asymptotic dimension of its underlying metric space. Finally, we will discuss its relation to the dynamic asymptotic dimension of groupoids introduced by Guentner, Willett and Yu and the (fine) tower dimension of topological dynamical systems introduced by Kerr.

On the Dichotomy of Paradoxical versus Amenable

Fernando Lledó

University Carlos III de Madrid and ICMAT, Madrid

E-mail address: flledo@math.uc3m.es

In this talk I will address the dichotomy between amenability and the existince of paradoxical decompositions in different mathematical contexts where a natural notion of dynamics is present. These include semigroups, algebras, metric spaces and operator algebras. Semigroups have singular dynamics in the sense that multiplication can drastically shrink the size of a finite subset in the semigroup. This fact forces to analyze again basic aspects in group theory like amenability, paradoxical decompositions or existence of invariant means. We will analyze these aspects and present some recent results in the context of inverse semigroups and their reduced C*-algebras.

References

[1] P. Ara, F. Lledó and D. Martínez, Amenability and paradoxicality in semigroups and C*-algebras, arXiv: math.OA/1904.13133

[2] P. Ara, K. Li, F. Lledó and J. Wu, Amenability and uniform Roe algebras, J. Math. Anal. Appl. **459** (2018) 686-716

A sheaf cohomology theory for C*-algebras

Martin Mathieu

Queen's University Belfast, University Rd, Belfast BT7 1NN, United Kingdom

E-mail address: m.m@qub.ac.uk

I shall report on our ongoing project with Pere Ara in which we develop a sheaf cohomology theory for general C^*-algebras. In particular, I will emphasize the role C^*-ringed spaces (the analogue of ringed spaces in Algebraic Geometry) play and the various ways to define the sheaf cohomology groups.

Some computer-assisted ring theory

Kevin C. O'Meara

University of Canterbury, Giles Ln, Canterbury CT2 7NZ, United Kingdom

E-mail address: staf198@uclive.ac.nz

Three topics will be discussed:

(1) The nearly 60 year-old Gerstenhaber Problem (GP) of whether Gerstenhaber's most surprising 1961 theorem (Annals of Math.) for two commuting matrices extends to three commuting $n \times n$ matrices A, B, C over a field F: must the (unital) subalgebra $F[A, B, C]$ of $M_n(F)$ generated by A, B, C have dimension at most n? (Easy examples show this fails for four or more.) Recently, I showed the GP is "decidable" in terms of Turing computability, whence ideally suited to a computer-generated counter-example. (I believe the answer to the GP is "no".)

(2) The recent formula by James Pascoe (University of Florida) for the dimension over the real or complex field K of an algebra generated by ***arbitrary*** (noncommuting) $n \times n$ matrices $X_1, X_2, \ldots, X_d$, and without involving any product $X_i X_j$ of the generators! Using mod p techniques, James and I have written a super-fast MATLAB program to compute this dimension over a number field K. This can then be used to determine algebra membership, for example.

(3) Briefly relating these computing techniques to the possibility of a computer-generated nonseparative regular ring. The important concept of separativity was first formulated in the work of Pere Ara, Ken Goodearl, Enrique Pardo, and myself in the mid 1990's.

Largest ideals in Leavitt path algebras

Mercedes Siles Molina

Departamento de Álgebra, Geometría y Topología, Universidad de Málaga, Av. de Cervantes, 2, 29016 Málaga, Spain

E-mail address: msilesm@uma.es

The aim of this talk is to identify some largest ideals in Leavitt path algebras: the largest locally left/right artinian (which is the largest semisimple one), the largest locally left/right noetherian without minimal idempotents, the largest exchange, and the largest purely infinite. This last ideal is described as a direct sum of purely infinite simple pieces plus purely infinite non-simple and non-decomposable pieces. The invariance under ring isomorphisms of these ideals is also studied.

Duality for rings constructed from groupoids

Aidan Sims

University of Wollongong, Northfields Ave, Wollongong NSW 2522, Australia

E-mail address: asims@uow.edu.au

A key tool in the study of C^*-algebras associated to étale groupoids is the theorem, due to Kumjian and Renault, that states that an étale groupoid can be reconstructed from the associated C^*-algebra and its diagonal subalgebra. I will discuss joint work with Ara, Bosa and Hazrat establishing an analogous similar result in the setting of Steinberg algebras, and discuss some of the applications that it has found so far.

Anti-elementarity for ranges of functors

Friedrich Wehrung

LMNO, CNRS Université de Caen, Caen , France

E-mail address: friedrich.wehrung01@unicaen.fr

This talk deals with the question of the *tractability* of the range of a functor $\Phi\colon \mathcal{A} \to \mathcal{B}$, for arbitrary categories $\mathcal{A}$ and $\mathcal{B}$. For example, if $\mathcal{B}$ consists of structures in a given first-order language (e.g., lattices, monoids, groups), tractability follows from having a first-order description. Following that convention, one can for example say that the class of all refinement monoids is tractable within the class of all monoids. A far less stringent definition of tractability arises if usual first-order logic is weakened by allowing conjunctions and disjunctions of arbitrary (possibly infinite) length, and quantifiers over strings of variables of length less than a given infinite cardinal λ; the corresponding logic is usually denoted by $\mathcal{L}_{\infty\lambda}$. For example, simple refinement monoids are $\mathcal{L}_{\infty\omega}$-definable (but not first-order) within monoids.

We present a new technique [3], extending the author's original work with Pierre Gillibert [2], that makes it possible to prove intractability results for various functors intervening in algebra. This technique lives in the world of categorical model theory (see for example [1]), and this is also where our concept of intractability, that we call *anti-elementarity*, comes from. Our key construction, mixing together products and directed colimits, is the one of *condensate*. Here is a sample of classes for which this makes it possible to prove anti-elementarity:

- the class of all lattices of finitely generated convex ℓ-subgroups of members of any class of ℓ-groups containing all Archimedean ℓ-groups;
- the class of all semilattices of finitely generated ℓ-ideals of members of any nontrivial quasivariety of ℓ-groups;
- the class of all semilattices of finitely generated two-sided ideals of von Neumann regular rings;
- the class of all semilattices of finitely generated submodules of modules;
- the class of all monoids encoding the nonstable K_0-theory of von Neumann regular rings, respectively C*-algebras of real rank zero.

The main underlying principle is that under quite general conditions, for a functor $\Phi\colon \mathcal{A} \to \mathcal{B}$, if there exists a (necessarily non-commutative) diagram $\vec{D}$ of $\mathcal{A}$, indexed by (say) a finite lattice, such that

- $\Phi\vec{D}^I$ is a commutative diagram for every set I,
- $\Phi\vec{D} \ncong \Phi\vec{X}$ for any commutative diagram $\vec{X}$ in $\mathcal{A}$,

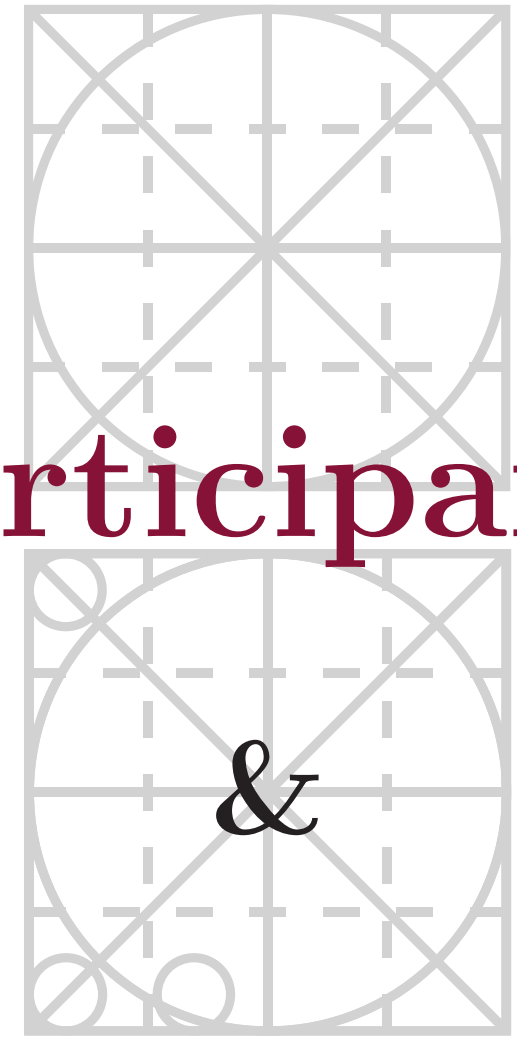
then the range of Φ is anti-elementary.

References

[1] Jiří Adámek and Jiří Rosický, *Locally Presentable and Accessible Categories*, London Mathematical Society Lecture Note Series, vol. 189, Cambridge University Press, Cambridge, 1994. MR 1294136 (95j:18001)

[2] Pierre Gillibert and Friedrich Wehrung, *From Objects to Diagrams for Ranges of Functors*, Lecture Notes in Mathematics, vol. 2029, Springer, Heidelberg, 2011. MR 2828735 (2012i:18001)

[3] Friedrich Wehrung, *From non-commutative diagrams to anti-elementary classes*, hal-02000602, preprint, 2019.



Participants & Collaborators

List of Participants

Ramon Antoine	Universitat Autònoma de Barcelona
Christian Bönicke	University of Glasgow
Joan Bosa	Universitat Autònoma de Barcelona
Ferran Cedó	Universitat Autònoma de Barcelona
Joan Claramunt	Universitat Autònoma de Barcelona
Guillermo Cortiñas	Universidad de Buenos Aires
Warren Dicks	Universitat Autònoma de Barcelona
Kenneth Ralph Goodearl	University of California
Roozbeh Hazrat	Western Sydney University
Dolors Herbera	Universitat Autònoma de Barcelona
Kang Li	Institute of Mathematics Polish Academy of Science
Fernando Lledó	Universidad Carlos III
Martin Mathieu	Queen's University
Kevin C. O'Meara	University of Canterbury
Francesc Perera	Universitat Autònoma de Barcelona
Kulumani Rangaswamy	University of Colorado Boulder
Mercedes Siles	Universidad de Málaga
Aidan Sims	University of Wollongong
Eduard Vilalta	Universitat Autònoma de Barcelona
Friedrich Wehrung	CNRS

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