

***Stability and instability of equilibrium solutions of Hamiltonian systems with  $n$ -degrees of freedom with a single resonance: the critical case and under the existence of an invariant ray***

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In this talk we present new results for the stability of one equilibrium solution of an autonomous analytic Hamiltonian system in a neighborhood of the equilibrium point with  $n$ -degrees of freedom under the existence of a single resonance. In the first part, we give information in the critical cases (i.e., the condition of stability and instability is not fulfilled), our Main Theorem generalizes several results existing in the literature. We show the applicability of our results to the stability of Hamiltonians of the type of cosmological models as in planar as in the spatial case. In the second part, we prove the instability of one equilibrium point in an autonomous Hamiltonian system with  $n$ -degrees of freedom under the existence of an invariant ray solution of the truncated Hamiltonian system up to order  $s$ . Application of our main result to the satellite problem is considered.

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