On the n-body problem on surfaces of revolution

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We explore the n-body problem, $n > 2$, on a surface of revolution with a general interaction depending on the pairwise geodesic distance. Using the methods geometric mechanics we determine a large set of properties. In particular, we show that Saari's conjecture fails on surfaces of revolution admitting a geodesic circle. We define homographic motions and, using the symmetries, prove that when the masses are equal, they form an invariant manifold on which the dynamics is reducible to a one-degree of freedom system. We also find that for attractive interactions, regular n-gon-shaped relative equilibria with trajectories located on geodesic circles typically experience a pitchfork bifurcation. Some applications are included, including motion on the 2-dimensional sphere, for which we show that the aforementioned bifurcation is associated with a change in stability.