

*A perturbed Hamilton-Hopf model: splitting, dominant harmonics and hidden harmonics*

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We consider a Hamiltonian with an equilibrium having a Hopf bifurcation for  $\nu = 0$  and becoming a complex saddle for  $\nu > 0$ . After normalization and scaling, with coefficients giving rise to compact invariant manifolds, we add a very simple perturbation depending on position and periodic in time, but which contains infinitely many terms. The perturbation is considered as a fixed function and the relevant parameter is  $\nu$ . The goal is to study the behavior of the system for small values of  $\nu$ .

The position along the unperturbed manifolds contains factors depending periodically on time and, with a suitable normalization, they have unit basic frequency. On the other hand the frequency of the perturbation can be chosen in an arbitrary way. The amplitudes of the coefficients of the harmonics appearing in the perturbation decrease in an exponential way.

We study the splitting of the manifolds produced by the perturbation, mainly using first order Poincaré-Melnikov approach. Due to the frequencies involved the splitting becomes a quasiperiodic function of two angles. After displaying some information on the nodal lines and its bifurcations, we look for the changes in the relevant harmonics in the splitting. That is, how the dominant harmonics in the splitting change as a function of  $\nu$ .

It turns out that these dominant harmonics are related to best approximants of the frequency of the perturbation. It can happen, depending on that frequency, that there exist harmonics associated to best approximants which never dominate the splitting functions. They are named “hidden” harmonics.

Why and when hidden harmonics associated to best approximants exist is studied theoretically. It depends on arithmetic properties of the perturbing frequency. This leads, in a natural way, to several open questions in number theory.

This is part of joint works with E. Fontich and A. Vieiro [1, 2].

#### REFERENCES

- [1] E. Fontich, C. Simó, and A. Vieiro, Splitting of the separatrices after a Hamiltonian-Hopf bifurcation under periodic forcing. Preprint, 2018.
- [2] E. Fontich, C. Simó, and A. Vieiro, On the “hidden” harmonics associated to best approximants due to quasi-periodicity in splitting phenomena. Preprint, 2018.