Applications of Conley-Zehnder index to elliptic stability for periodic orbits in Hamiltonian and Lagrangian systems

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Roughly speaking, the Conley-Zehnder index measures the number of half windings of a fundamental solution for a periodic linear Hamiltonian system. This index, and its closely related cousin the Morse index for the equivariant action functional, can be used to give non perturbative arguments for linearized stability and instability for families of periodic orbits in Hamiltonian systems. Our first example of this idea highlights the roll of mountain pass critical points in the forced pendulum equation. We obtain a condition, earlier expressed by R. Ortega using different arguments, on the parameters of the system which guarantee elliptic stability for the mountain pass critical points. Our next example illustrates this construction for finding families of elliptic periodic orbits in the Sitnikov problem. This system describes the dynamics of a negligble mass oscillating on a vertical axis of symmetry through the center of mass for two massive bodies moving in the plane on Keplerian ellipses.