

Tensor triangular geometry via tensor abelian categories

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There is a rich interplay between a compactly generated triangulated category and the abelian category of modules over its compact objects. These categories are linked by the restricted Yoneda functor, which sends an object to the functor it represents restricted to the compact objects, i.e. it captures how the compacts see that object. The link between these categories has proved very fruitful, leading to the theory of purity in triangulated categories and to advances in our understanding of localization theory. I will introduce this setup, with the additional presence of a compatible symmetric monoidal structure on the triangulated category. One then gets an induced symmetric monoidal structure on the module category, and the restricted Yoneda functor preserves the tensor product. This framework has been recently used, by Balmer, Krause, and myself, to make progress on two open questions in the theory of tt-geometry: what is the spectrum of a compactly generated tensor triangulated category, and how can one define and construct tensor triangular analogues of residue fields? The goal of the course is to introduce these questions (which are still open!), and explain how one can use the corresponding abelian category to shed some light on them.