Periodic orbits and invariant tori in ideal fluid flows

Daniel Peralta-Salas, ICMAT Madrid
E-mail address: dperalta@icmat.es.

Ideal fluid flows are modelled by the Euler equations, a complicated system of non-linear PDEs where the unknowns are a non-autonomous vector field representing the velocity of the fluid and a time-dependent scalar function representing the pressure.

The integral curves of the velocity field are the fluid particle paths, so the analysis of these orbits from the dynamical systems viewpoint is relevant to understand the fluid motion. The goal of this course is to present some results on the trajectories of autonomous vector fields that are time-independent (or stationary) solutions of the 3D Euler equations. This (huge) family of vector fields is known as Euler fields.

The course consists of four lectures. In Lecture 1 I will review Arnold's structure theorem on the integrability of typical Euler vector fields. In the second Lecture I will show that any analytic and non-vanishing Euler field on the 3-dimensional sphere has a periodic orbit. The last two lectures will be devoted to the study of a particularly relevant class of Euler fields, known as Beltrami fields, which are eigenfunctions of the curl operator. I will introduce Arnold's conjectures in this setting, and will present two realization theorems for Beltrami flows in Euclidean space. The first theorem shows that, for any locally finite union of pairwise disjoint (possibly knotted and linked) closed curves, there exists a Beltrami field with a set of hyperbolic periodic orbits diffeotopic to this set of curves. The second theorem claims that, for any set of finitely many pairwise disjoint (possibly knotted and linked) embedded tori, there exists a Beltrami field with a set of invariant tori diffeotopic to the aforementioned set.

Time permitting, I will discuss some applications of these results to analyze the existence of bifurcations in time-dependent solutions to the 3D Navier-Stokes equations.