Avalanches on networks: From phase transitions to explosive contagion and immunisation

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The behaviour of many systems can be described in terms of a (typically large) number of elementary units and their interactions. Examples range from solids to the human body and society. A small change in the environment of such systems may lead to avalanche-like responses of the system associated with sudden changes of state in a large number of elementary units. In the last decades, network models have become essential abstractions of such systems. They are applications of graph theory in which a set of vertices (or nodes) represent elementary units and a set of edges (or links) gives the interaction between them. In this talk, I will present prototype network models for avalanches in solid-solid phase transitions [1] and capillary condensation in porous media [2] as well as for avalanche-like phenomena in social contagion [3] and immunisation strategies [4]. I will then focus on spin models with quenched disorder which play a central role in understanding avalanche dynamics. In particular, I will present exact results for the zero-temperature random-field Ising model on relatively simple network topologies (q-regular graphs [5,6] with one and two layers [7]). The solutions, which are based on recursive relations and generating function techniques, do not only give the magnetisation and size of avalanches but allowed us to obtain an expression for the correlation function and investigate time-dependent properties such as the avalanche durations, spectrum and pulse shape. In addition, the analytical approach allows the microscopic criteria for the occurrence of an infinite avalanche in a q-regular graph to be determined. For instance, this explains why an infinite avalanche can occur in q-regular graphs with coordination number $q>3$. In contrast, a “weak” infinite avalanche is possible on a bilayer q-regular graph with $q=3$.

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