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The Breiman Conjecture.

Abstract: Let $Y, Y_1, Y_2, \ldots$ be positive, nondegenerate, i.i.d. G random variables, and independently let $X, X_1, X_2, \ldots$ be i.i.d. F random variables. Breiman (1965) conjectured that if X is nondegenerate, $\mathbb{E}\,|X| < \infty$ and $\mathbb{T}_n = \sum X_i Y_i / \sum Y_i \to_d T$, where T is nondegenerate, then necessarily $\overline{G} = 1 - G$ is regularly varying at infinity with index $0 \le \beta < 1$, written $G \in D\,(\beta)$. In this talk we discuss the recent progress of Péter Kevei and David Mason towards resolving this conjecture. We have shown that whenever for some $F \in \mathcal{F}$, in a specified class of distributions $\mathcal{F}$, $\mathbb{T}_n$ converges in distribution to a nondegenerate limit then necessarily $G \in D\,(\beta)$ with $0 \le \beta < 1$. The class $\mathcal{F}$ contains the distributions of nondegenerate X with a finite second moment, as well as those of X in the domain of attraction of a stable law with index $1 < \alpha < 2$. Our results will appear in the *Proceedings of the AMS*. We shall also discuss the limiting distributional behavior of these self-normalized sums along subsequences and their Lévy process analogs.