FROBENIUS ALGEBRAS OF STANLEY-REISNER RINGS

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Let R be a commutative ring of characteristic $p > 0$ and M an R-module. Let $\mathcal{F}(M)$ be the ring of Frobenius operators of M as defined by G. Lyubeznik and K. Smith in 2001. M. Katzman gave in 2010 an example showing that this algebra may not be finitely generated. The ring R in Katzman's example is a non-Cohen-Macaulay quotient of a formal power series ring in three variables by a square-free monomial ideal, and the module M the injective envelope E_R of the residue field of R.

In this talk I will explain how to extend Katzman's idea to study the ring of Frobenius operators of the injective envelope of the residue field of any (complete) Stanley-Reisner ring R over a field of prime characteristic $p > 0$. It is possible to give a precise description of $\mathcal{F}(E_R)$ that shows that this algebra can only be principally generated or infinitely generated depending on the minimal primary decomposition of the defining monomial ideal of R. Examples will be given showing that one may find both non Cohen-Macaulay ideals with principally generated Frobenius algebra and Cohen-Macaulay ideals with infinitely generated Frobenius algebra. In fact, one may give a purely combinatorial characterization of those Stanley-Reisner rings with principally generated Frobenius algebra, due to J. Àlvarez-Montaner and K. Yanagawa in 2014. As an application one can see that independently of the finite or non finite character of the Frobenius algebra $\mathcal{F}(E_R)$, its Matlis dual algebra $\mathcal{C}(R)$ of Cartier operators is always gauge bounded, a notion introduced by M. Bilckle in 2013, which implies that the set of F-jumping numbers of the corresponding generalized test ideals is always a discrete set.

The talk is based on joint work with Josep Àlvarez Montaner and Alberto Fernández Boix.