Brauer-Thrall for totally reflexive modules

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Let $(R, \mathfrak{m}, k)$ be a local ring. An R-module M is *totally reflexive* provided there is a doubly-infinite exact sequence

$$\mathbf{F} = \quad \cdots \to F_2 \to F_1 \to F_0 \to F_{-1} \to F_{-2} \to \dots$$

such that each F_n is a finite-rank free module, $M \cong \mathrm{Coker}(F_1 \to F_0)$, and the dual sequence $\mathrm{Hom}_R(\mathbf{F}, R)$ is also exact. Equivalently, M is finitely generated, M is reflexive, and both $\mathrm{Ext}^n_R(M, R)$ and $\mathrm{Ext}^n_R(M^*, R)$ vanish for all $n > 0$. If R is Gorenstein, the totally reflexive modules are just the maximal Cohen-Macaulay modules. If R is *not* Gorenstein and has a non-free totally reflexive module, it is known [Christensen, Piepmeyer, Striuli, Takahashi, 2009] that R must have infinitely many indecomposable totally reflexive modules. I will describe some recent work, with Hamid Rahmati and Janet Striuli, which exploits the $\mathrm{End}_R(M) - \mathrm{End}_R(N)$-bimodule structure on $\mathrm{Ext}^1_R(N, M)$, to build, under suitable hypotheses, infinitely many non-isomorphic indecomposable modules X fitting into an exact sequence $0 \to M \to X \to N \to 0$. I will show how to use this result to obtain infinitely many indecomposable totally reflexive modules of fixed large multiplicity (again, under suitable hypotheses). An interesting feature of the construction is that it works even when the residue field k is finite.