An alternative perspective on flatness of modules

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Abstract

Given modules ${}_RM$ and A_R, ${}_RM$ is said to be absolutely A_R-pure if $A \otimes M \to A \otimes B$ is a monomorphism for every extension ${}_RB$ of ${}_RM$. For a module A_R, the absolutely pure domain of A_R is defined to be the collection of all modules ${}_RM$ such that ${}_RM$ is absolutely A_R-pure. As an opposite to flatness, a module A_R is said to be f-indigent if its absolutely pure domain is smallest possible, namely, consisting of exactly the fp-injective modules. Properties of absolutely pure domains and of f-indigent modules are studied. In particular, the existence of f-indigent modules is determined for an arbitrary rings. For various classes of modules (such as finitely generated, simple, singular), necessary and sufficient conditions for the existence of f-indigent modules of those types are studied. Furthermore, f-indigent modules on commutative Noetherian hereditary rings are characterized.

Keywords

Whitehead test modules; f-test modules; (p-,f-)indigent modules; flat modules; fully saturated rings.

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