

Choiceless cardinals and I_0

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It is a well-known open problem whether Kunen's inconsistency theorem can be proved without appealing to the Axiom of Choice. Until recently, however, it was not even known whether the assumption of the existence of an elementary embedding from V to V was particularly strong in the choiceless context, let alone inconsistent: for example, it was unknown whether it implied the consistency of ZFC along with the existence of a superstrong cardinal. In his work on the HOD Conjecture, Woodin obtained the consistency strength lower bound of ZFC along with the existence of a proper class of extendible cardinals (and more). In fact he showed that if there is a proper class of extendible cardinals it is possible to *force choice* while preserving all extendible cardinals. We will discuss recent results that come very close to improving this lower bound to $\text{Con}(\text{ZFC} + I_0)$.