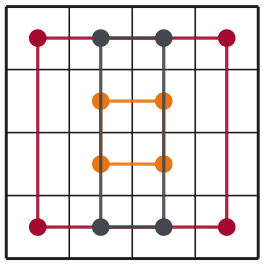
ACTIVITY 6 · CHAPTER 08 · THE (ALMOST) MAGIC SQUARE

Your own (almost) magic square



On the Passion façade of the Sagrada Família there are sixteen numbers that add up to 33 along rows, columns and diagonals. The sculptor Josep Maria Subirachs put them there; they are not Gaudí's. Here you will see why his square can't be fully magic, and you will make one with your own date of birth.

Name _____ Class _____ Date _____

TIME	50 minutes
GROUPS	On your own or in pairs
YOU NEED	Pencil, colours and, if you need one, a calculator.

A Subirachs's square

A1 Add up each row, each column and both diagonals. Write the totals in the grey boxes, and the diagonals in the dashed ones.

A2 There are many more ways to pick four cells that add up to 33: the four corners, the four in the middle... Mark them by joining the cells with a coloured line, like the drawing at the top. There are 88 in all. How many can you find? .....

1	14	14	4	
11	7	6	9	
8	10	10	5	
13	2	3	15	

A3 Which numbers appear twice? _____ Which numbers from 1 to 16 are missing? _____

A4 A "real" 4 × 4 magic square uses the numbers 1 to 16, each exactly once. What do all sixteen add up to? (Hint: pair 1 with 16, 2 with 15... How many pairs are there, and what does each pair add up to?)

_____

If that total is shared out over four rows with equal sums, what must each row add up to? _____ Why couldn't Subirachs get to 33 without repeating numbers?

_____

_____

B Your birthday square

Write your date of birth as four two-digit numbers: A, the day; B, the month; C, the first two digits of the year; and D, the last two. For example, 15 March 2013 gives A = 15, B = 3, C = 20 and D = 13. Then fill in the empty square following the template.

The template

A	B	C	D
D + 1	C − 1	B − 3	A + 3
B − 2	A + 2	D + 2	C − 2
C + 1	D − 1	A + 1	B − 1

Example: 15-03-2013

15	3	20	13
14	19	0	18
1	17	15	18
21	12	16	2

Your square

B1 Check that the rows, columns and diagonals all add up to the same number. What is it? ..... Compare it with A + B + C + D. .....

B2 Do the four corners add up to the same number? And each of the four 2 × 2 squares in the corners?

B3 Why does it work? Look at the second row: D + 1, C − 1, B − 3, A + 3. What happens to the +1 and −1, and to the −3 and +3, when you add it all up? Do the same with the first column.

B4 You might get a zero or a negative number (if you were born in January, B − 3 = −2). That's fine: the square still works. Like Subirachs's, yours isn't a real magic square either. Why not?

GOING FURTHER

Make the square for Gaudí's birthday: he was born on 25 June 1852. What do the rows add up to? Are any numbers repeated? And another challenge: in the square Albrecht Dürer engraved in 1514, the bottom row hides the year of the engraving (15 14). Where is your date in your square?

Your own (almost) magic square

SOLUTIONS AND NOTES

AIMS

Practise calculation, use Gauss's pairing trick, understand why terms cancel in an expression and reason about why something is impossible.

MATHS TOPICS

Number · Algebra (expressions and equivalence).

TIPS

- In part A, groups can hunt for combinations and pool them on the board.
- In part B, anyone who would rather not use their own date can pick a relative's or a made-up one.

WHERE SUBIRACHS'S SQUARE COMES FROM

From Dürer's, which adds up to 34. Subirachs rotated it half a turn and took 1 away from four cells, one in every row, column and diagonal, so each line loses exactly 1. 33 is the age tradition gives Christ at his death.

THE BIRTHDAY SQUARE

The template is often called "Ramanujan's magic square", because it is usually shown with the birth date of the Indian mathematician Srinivasa Ramanujan, 22-12-1887.

SOLUTIONS

- A1 They all add up to 33.
- A2 The corners (1 + 4 + 13 + 15), the centre (7 + 6 + 10 + 10), the four 2 × 2 corner squares (1 + 14 + 11 + 7, 14 + 4 + 6 + 9, 8 + 10 + 13 + 2, 10 + 5 + 3 + 15), the middle cells of the top and bottom rows (14 + 14 + 2 + 3), the outer cells of the two middle rows (11 + 9 + 8 + 5)… Counting cells rather than values, there are 88 groups of four. Counting every group of three to seven cells (the 88 included) gives 310, the figure the Sagrada Família quotes.
- A3 10 and 14 appear twice; 12 and 16 are missing.
- A4 8 pairs of 17: 136. 136 ÷ 4 = 34. For 33 the total would have to be 132, and with the numbers 1 to 16 used once each it is always 136.
- B1 Always A + B + C + D. In the example, 51.
- B2 Yes: the corners and the four 2 × 2 squares, and the four central cells too.
- B3 Every row, column and diagonal has A, B, C and D once each, plus numbers that add up to zero (second row: +1 − 1 − 3 + 3; first column: +1 − 2 + 1).
- B4 Numbers can repeat, and they are not 1 to 16.
- Gaudí 25 + 6 + 18 + 52 = 101, and no number repeats.

To follow up: chapter 08 at crm.cat/les-matematiques-de-gaudi, with Subirachs's and Dürer's squares to play with; the Sagrada Família blog article "The magic square on the Passion façade"; and the National Gallery of Art (Washington) page for "Melencolia I".

FURTHER READING

- Weisstein, E. W. "Dürer's Magic Square" and "Magic Square". *MathWorld*, Wolfram Research.

The full bibliography, with links, is at crm.cat/les-matematiques-de-gaudi.