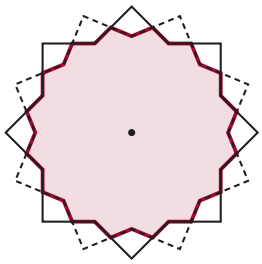
ACTIVITY 5 · CHAPTERS 06 AND 07 · THE COLUMN AND THE MODULE

The climbing star



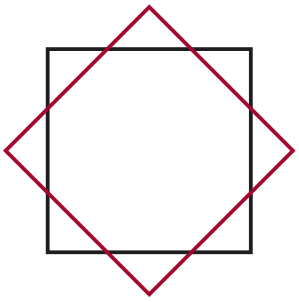
The columns of the Sagrada Família's central nave start as an eight-pointed star and become almost round as they rise. They do it by turning right and left at the same time. With two sheets of tracing paper you can see how.

Name _____ Class _____ Date _____

TIME	50 minutes
GROUPS	In pairs
YOU NEED	Two sheets of tracing paper (or the template on page 3 printed twice on tracing paper), a drawing pin, an eraser, coloured pencils, a ruler, a 16 cm strip of paper and scissors.

A Two turning stars

1. Trace the star and the ring of marks on page 3 onto two sheets of tracing paper.
2. Lay one sheet on the other, with the stars exactly on top of each other, and push the pin through the centre into the eraser.
3. Turn the top sheet until its mark 0 sits over mark 1 on the bottom sheet.
4. Colour in the part that is inside both stars at once.



two squares, one turned 45°

A1 How many points does the figure you coloured have? .....

A2 Neighbouring marks are 22.5° apart. Check: there are ..... marks in a full turn, and 360 ÷ ..... = ...

A3 Now turn the top sheet one more mark (45° in total). What happens to the common part?

.....

A4 The column does something similar, but both stars turn at once, one to the right and the other to the left. The cross-section is the part they share, and at the top of the first section it already has sixteen points. If the process were repeated with the sixteen-pointed figure, how many points would you get? And the time after that? What happens to the shape when there are lots and lots of points?

.....

.....

B Sections that get shorter

In the eight-pointed column the first section is 8 metres tall. Each following section is half as tall as the one before and doubles the points again.

Section	Height of section	Total height	Points at the top
1	8 m	8 m	16
2			
3			
4			
5			

B1 If you could carry on the table forever, how high would the total get?

B2 In the real column the shaft ends after the third section, and the capital sits on top. How tall is the shaft, and how many points does it have at the top?

B3 Cut the 16 cm strip in half. Cut the piece you have left in half, and so on. Lay the cut pieces in a row: do they ever reach 16 cm?

B4 With fractions: 1/2 + 1/4 + 1/8 + 1/16 = _____. And if you add 1/32? _____ How much is left to reach 1? _____

C Columns you can count

The Sagrada Família has columns with 6, 8, 10 and 12 points. The number of points decides almost everything: the diameter is that many twelfths of 210 cm, and the first section is that many metres tall.

Points	Diameter	1st section	2nd section	3rd section	Shaft
6	105 cm	6 m	3 m	1.5 m	10.5 m
8		8 m			14 m
10					
12	210 cm				

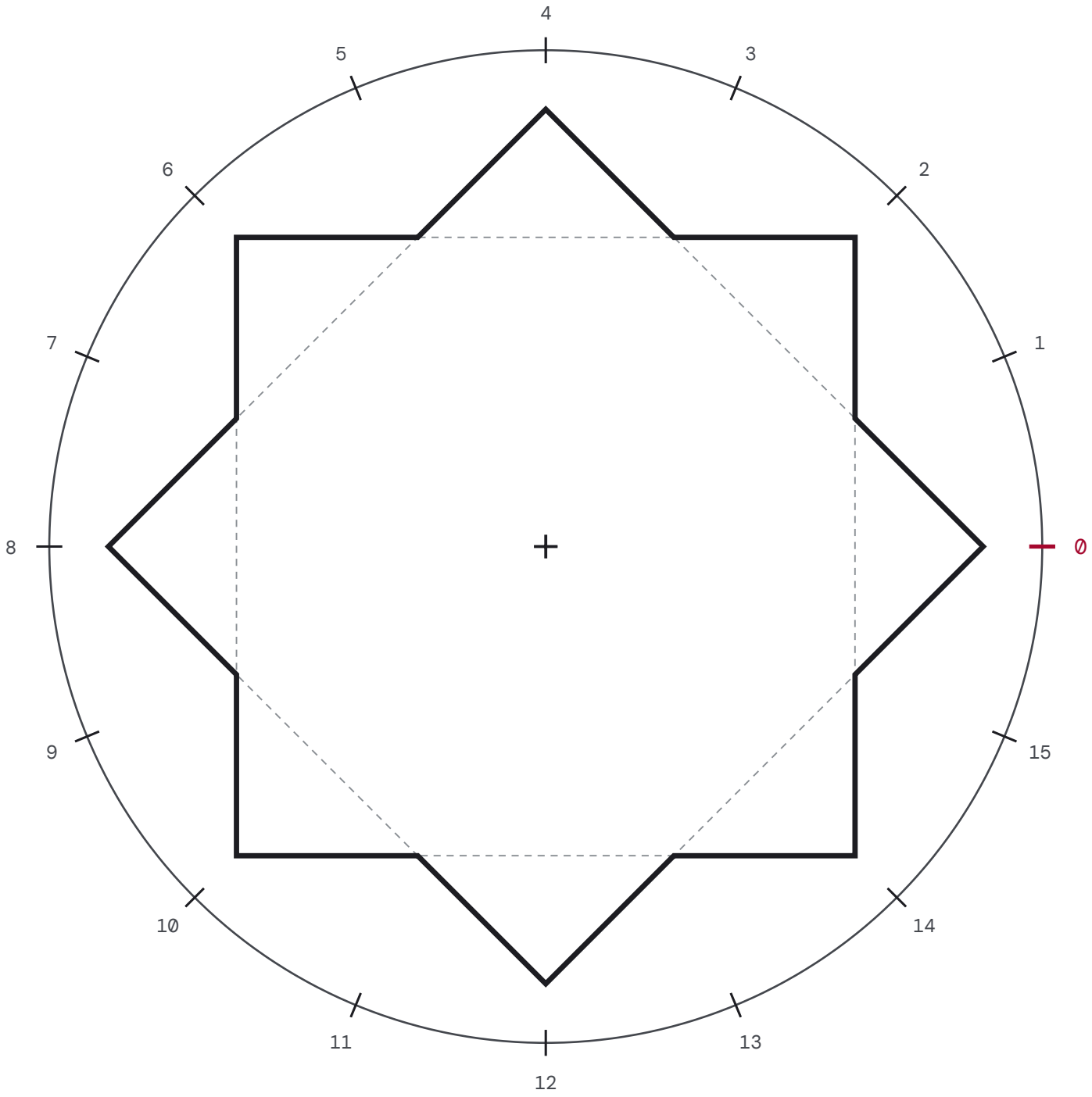
C1 How many centimetres of diameter go with each point? _____ cm

C2 Compare the diameter with the height of the shaft. How many times taller is it? _____

C3 The four twelve-pointed columns stand at the crossing, under the central tower, the tallest one. Why do you think they are the thickest?

Print this page twice on tracing paper, or trace it twice.

The eight-pointed star · trace it twice



marks every 22.5° · mark 0 sits on a point

The climbing star

SOLUTIONS AND NOTES

AIMS

Explore rotations and symmetry, discover an infinite sum with a finite result, work with fractions and recognise directly proportional quantities.

MATHS TOPICS

Geometry (rotation and symmetry) · Number (fractions) · Algebra (proportion).

TIPS

- Many laser printers take tracing paper. Otherwise trace the template or use acetate sheets.
- The eraser under the pin acts as an axle and protects the desk.
- In the real column the outline of the star is made of parabolic arcs rather than straight lines, but the process is the same.

WHY 22.5°

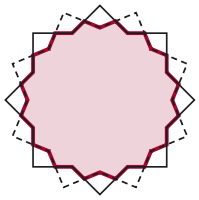
Any turn between 0° and 45° gives a common part with sixteen points, but only at 22.5°, half a step of the star, are they all equal. At 45° the two stars match again. So for the cross-section to have sixteen equal points, the two stars must end the first section 22.5° apart.

THE COLUMNS

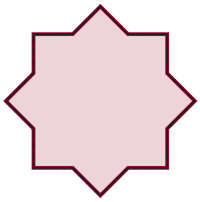
The twelve-pointed columns are the four at the crossing, in red porphyry; the ten-pointed ones, under the Evangelists' towers, are in black basalt.

To follow up: chapters 06 and 07 at crm.cat/les-matematiques-de-gaudi, where you can climb the column and watch its cross-section, and count the basilica's heights in 7.5 m modules; the Sagrada Família blog article "Sagrada Família columns: the geometry, mechanics and materials of a stone forest"; and the video "La columna de doble gir" (in Catalan) at educa.sagradafamilia.org.

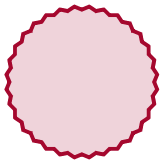
SOLUTIONS



22.5° turn: 16 equal points



45° turn: they match again



next section: 32 points

- A1 16.
- A2 16 marks; 360 ÷ 16 = 22.5°.
- A3 The stars match again: the common part is once more an eight-pointed star.
- A4 32, then 64… The shape gets closer and closer to a circle.

Section	2	3	4	5
Height	4 m	2 m	1 m	0.5 m
Total	12 m	14 m	15 m	15.5 m
Points	32	64	128	256

- B1 Up to 16 m, without ever reaching it.
- B2 14 m and 64 points. The remaining stretches are hidden inside the capital, an ellipsoid knot, and the whole column is 16 m tall, twice the number of points.
- B3 No: the piece you haven't cut is always missing.
- B4 15/16; 31/32; 1/32 is left.

Points	Diameter	Sections	Shaft
8	140 cm	8 + 4 + 2 m	14 m
10	175 cm	10 + 5 + 2.5 m	17.5 m
12	210 cm	12 + 6 + 3 m	21 m

- C1 17.5 cm per point.
- C2 10 times: 14 m is 1,400 cm, and 1,400 ÷ 140 = 10. It works for every column, and it is the rule the Sagrada Família itself gives: the shaft is ten times the inner diameter.
- C3 Because they carry more weight.

FURTHER READING

- Bonet i Armengol, J. (2000). *L'últim Gaudí = The Essential Gaudí*. Barcelona: Pòrtic. In Catalan and English.
- Burry, M. (ed.) (2007). *Gaudí Unseen: Completing the Sagrada Família*. Berlin: Jovis.

The full bibliography, with links, is at crm.cat/les-matematiques-de-gaudi.