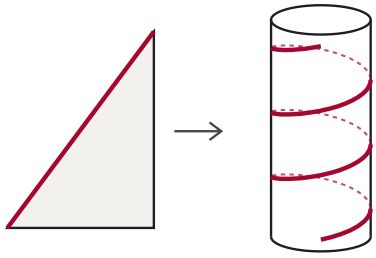
ACTIVITY 4 · CHAPTER 05 · LIVING CURVES

A straight line that coils

Cut out a paper triangle and roll it around a tube: its slanted edge turns into a helix, the curve of springs and of spiral-staircase handrails.

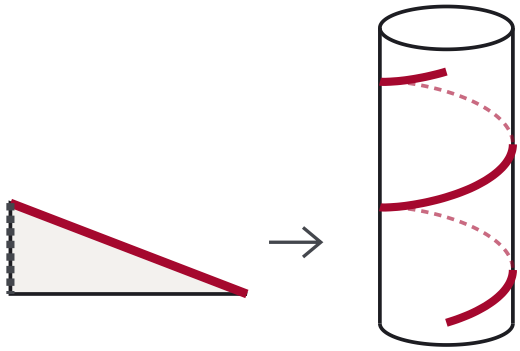


Name _____ Class _____ Date _____

TIME	50 minutes
GROUPS	In pairs
YOU NEED	The template on page 3, a cardboard tube (from kitchen roll or toilet roll), scissors, sticky tape and a ruler.

A From triangle to helix

1. Cut out triangle 1 and the measuring strip. The long slanted edge of the triangle is already coloured.
2. Tape the short side of the triangle (the 5 cm one) along the tube, from top to bottom rather than around it.
3. Roll the triangle tightly around the tube and tape down the tip.



the triangle rolls up and its edge makes a helix

A1 Wrap the strip around the tube and measure its circumference: C = _____ cm

A2 How many times does the triangle go round the tube? Divide: 26 ÷ C = _____

A3 How far does the coloured line rise in each turn? Use a proportion: if it rises 5 cm over a width of 26 cm, over C cm it rises _____ cm. Now measure it on the tube. Does it match?

A4 Repeat with triangle 2, which is twice as tall. How far does it rise in each turn now? _____ cm

A5 Look at the tube from the side, with your eyes level with it, and draw the part of the line you can see. Now imagine the tube is made of glass and add the hidden part as a dashed line. What shape do the two parts make together?

A6 Photographed from above, a spiral staircase looks like a flat spiral, like a sea snail's shell (there is a photo in chapter 05 of the web page). If the staircase is a helix, why do the lower turns look smaller in the photo?

B An imaginary spiral staircase

Each step on this staircase rises 18 cm and turns 24°.

B1 How many steps make one full turn? (A full turn is 360°.) .....

B2 How much does the staircase rise in one turn? ..... cm

B3 A tower is 54 metres tall. How many steps do you climb to reach the top? How many turns does the staircase make?

B4 If you could unroll the staircase wall like the paper triangle, what shape would the handrail be? Draw the unrolled wall, with the handrail and a few steps.

HELIX OR HELICOID?

A helix is a line, like the edge of the triangle. A helicoid is a surface: the one swept by a straight line that turns and rises at the same time, like the ramp in a car park. Sort these: a spring, a car-park ramp, the handrail of a spiral staircase, a spiral slide, a DNA molecule.

Helix	
Helicoid	

tape this side to the tube

Triangle 2

26 × 10 cm

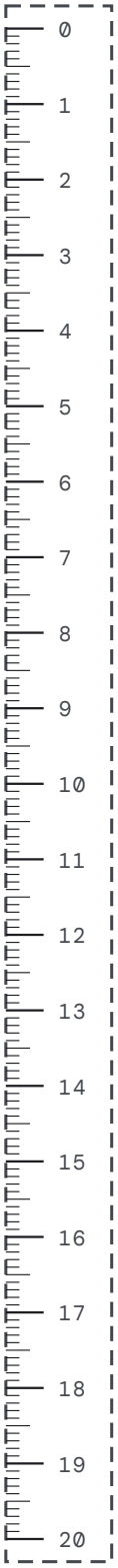
26 cm

26 cm

Triangle 1

26 × 5 cm

tape this side to the tube



measuring strip (cm)

Triangles to roll up and a measuring strip

Cut out both triangles and the strip. The thick edge is the one that will become a helix. The dotted side is the one you tape to the tube.

A straight line that coils

SOLUTIONS AND NOTES

AIMS

Connect a flat shape with the one it makes when rolled up, calculate with proportions, measure circumferences and use the degrees in a full turn.

MATHS TOPICS

Measurement · Number (proportion) · Geometry.

TIPS

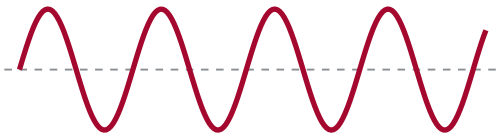
- Toilet-roll and kitchen-roll tubes are usually 12 to 14 cm round.
- If the triangle slips, start from the short side and roll the tube along the table.
- Part B needs no materials, only calculation.

GAUDÍ AND HELICES

Gaudí uses helices in the spiral staircases of the towers, in columns and in chimneys. The spiral you see when you photograph a spiral staircase from above is a perspective effect, not the logarithmic spiral of shells, even though it looks like one.

SOLUTIONS

- A1 Between 12 and 14 cm, depending on the tube.
- A2 26 ÷ C: about two turns.
- A3 5 × C ÷ 26. With C = 13 cm, 2.5 cm per turn.
- A4 Twice as much: with C = 13 cm, 5 cm per turn. The rise per turn is proportional to the height of the triangle.
- A5 A wave: a sine curve. Seen from the side, the helix $x = R\cos t, z = c\,t$ is the graph of a cosine. On the tube they only see the front half of each turn: slanted arcs from one edge to the other, one per turn.



- A6 Perspective: things further away look smaller.
- B1 360 ÷ 24 = 15 steps.
- B2 15 × 18 = 270 cm, that is, 2.70 m.
- B3 5,400 ÷ 18 = 300 steps, and 300 ÷ 15 = 20 turns.
- B4 A slanted straight line, like the edge of the triangle.
- Helix? Helix: the spring, the handrail and DNA (two helices, in fact). Helicoid: the ramp and the slide.

To follow up: chapter 05 at crm.cat/les-matematiques-de-gaudi, where the helix can be viewed from the side (a wave) or from above (a spiral).

FURTHER READING

- Alsina, C., Gómez Serrano, J. and Faulí, J. (2004). "La geometria i la mecànica a l'obrador de Gaudí". *Revista de Tecnologia*, 1, 12–27. In Catalan.
- Giralt-Miracle, D. (ed.) (2002). *Gaudí. Exploring Form*. Barcelona: Lunwerg.

The full bibliography, with links, is at crm.cat/les-matematiques-de-gaudi.