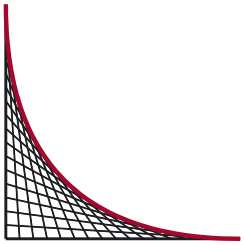
ACTIVITY 3 · CHAPTER 04 · RULED SURFACES

Curves made of straight lines



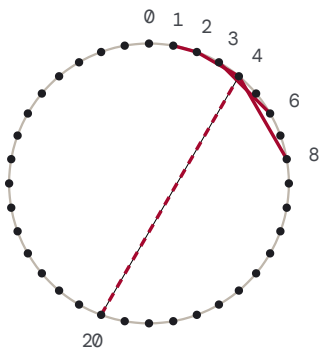
In this activity you will only draw straight lines, with a ruler. The curves will appear on their own. Gaudí used this to build curved shapes out of straight pieces, like the skylights in the vaults of the Sagrada Família.

Name _____ Class _____ Date _____

TIME	Part A, about 30 minutes. Part B, a 50-minute lesson.
GROUPS	Part A, on your own. Part B, in pairs.
YOU NEED	Ruler, coloured pencils, the templates on pages 3 and 4, card, glue, scissors and about 2 m of elastic thread (the kind used for bracelets).

A Straight lines that make curves

1. On the template on page 3 there is a right angle with its sides numbered 1 to 10. With the ruler, join the 1 on one side to the 1 on the other, 2 to 2, and so on up to 10. The first line is already drawn.
2. On the "× 2" circle there are 36 points numbered 0 to 35. Join each number to its double: 1 to 2, 2 to 4, 3 to 6… When the double goes past 35, subtract 36: 20 goes with 40 − 36 = 4.
3. On the "× 3" circle, do the same with the triple: 1 to 3, 2 to 6… Subtract 36 as many times as you need.



how "× 2" starts: 1 to 2, 2 to 4, 3 to 6, 4 to 8… and 20 to 4

A1 In the angle you have drawn ten straight lines. What shape appears?

A2 If each side had 20 marks instead of 10, would the curve look smoother or rougher? Why?

A3 What figure appears on the "× 2" circle? It is called a cardioid. Where do you think the name comes from?

A4 How many tips does the "× 3" figure have? And the "× 2" one? Guess how many tips the "× 4" figure would have.

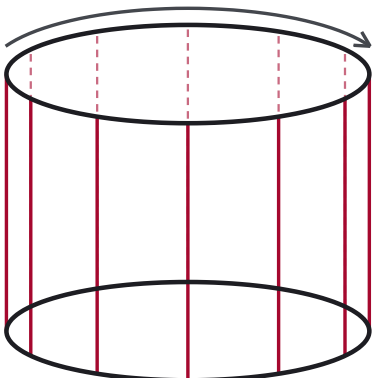
B A thread hyperboloid

1 Stick page 4 onto card and cut out the bottom circle and the top circle. Make a small notch, 3 mm deep, at each mark.

2 Tie the elastic thread to notch 1 on the bottom circle. Take it up to notch 1 on the top circle, over to notch 2, down to notch 2 on the bottom circle, and so on up to 12. Tie off the end.

3 A partner holds the bottom circle on the table. You hold the top one about 12 cm higher, notch above notch. You have a cylinder made of straight threads.

4 Turn the top circle by one notch, two, three... up to six. Each time, measure the width of the waist at its narrowest point with the ruler, and write it in the table.



the thread goes up and down through the notches

Notches turned	0	1	2	3	4	5	6
Degrees	0	30	60	90	120	150	180
Waist measured (cm)							
Waist from drawing (cm)							

B1 Check by drawing. On the circle you don't cut out, on page 4, draw the line from point 1 to point 4: it is what one thread looks like from above when you turn three notches. Mark its midpoint, measure its distance from the centre and multiply by 2. Do it for each turn (1 to 2, 1 to 3...) and fill in the last row of the table.

B2 Do the last two rows of the table match? What happens when you turn six notches (180°)?

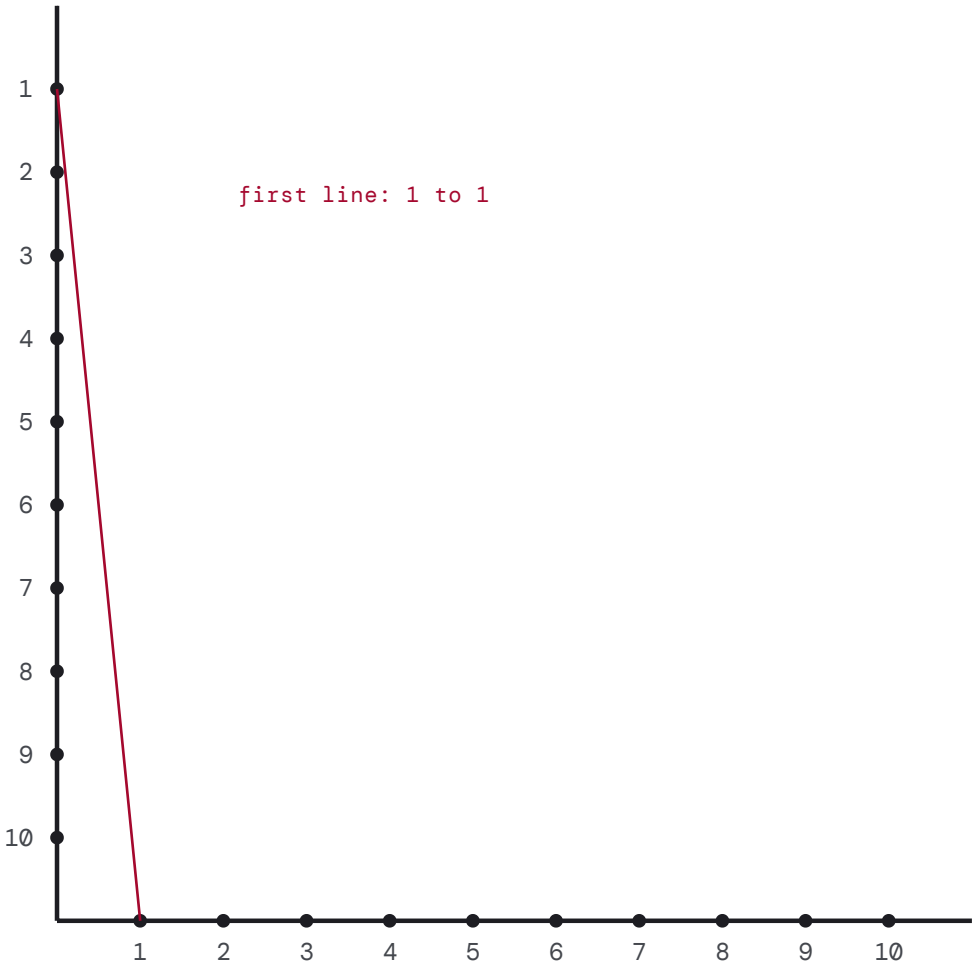
B3 Why do the drawing and the model give the same result? Hint: look at the model from above.

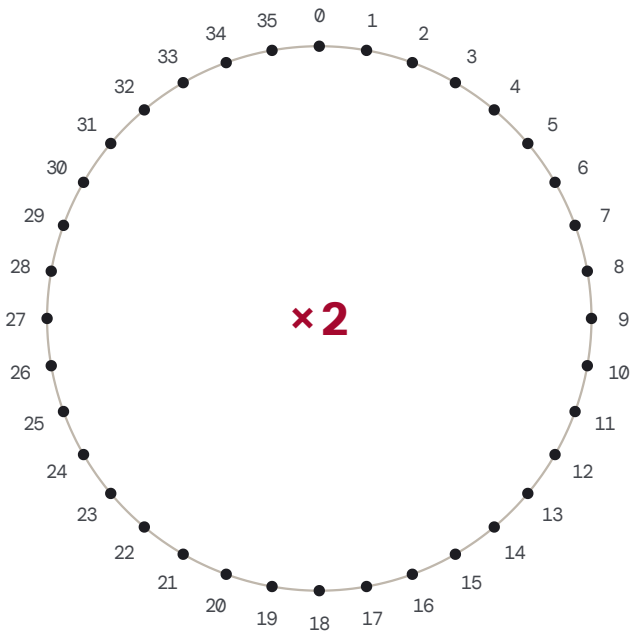
GOING FURTHER

Try "× 4", "× 5" and "× 6" on circles you draw yourself and check whether your guess in question A4 works. And look at your hyperboloid: it is all curves, yet it is made only of straight threads. That is why it can be built with bars, formwork planks or a taut thread.

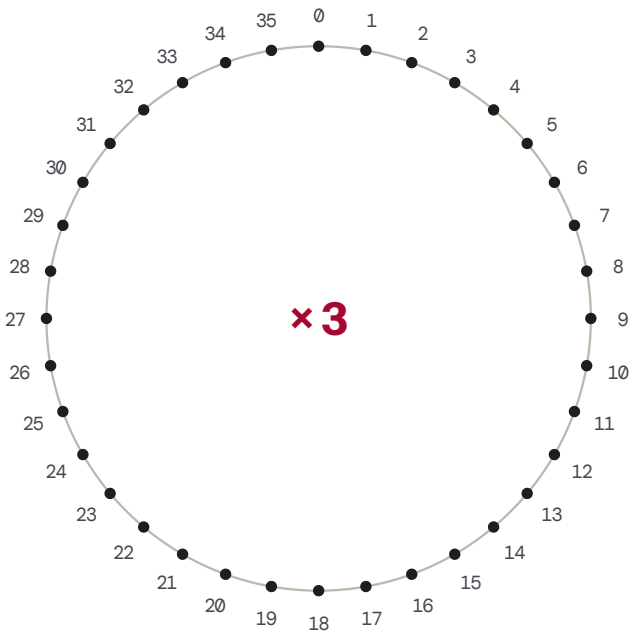
Template for part A. Nothing to cut out.

Straight lines that make a curve





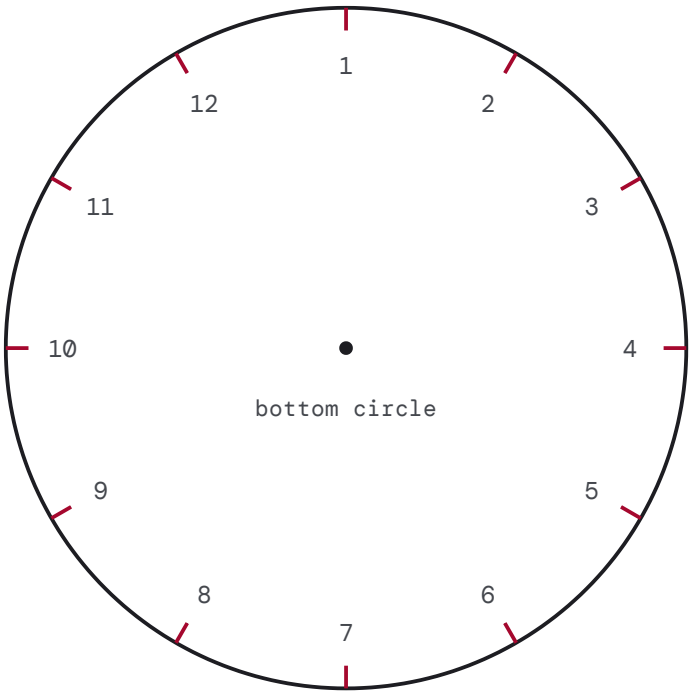
join each number to its double

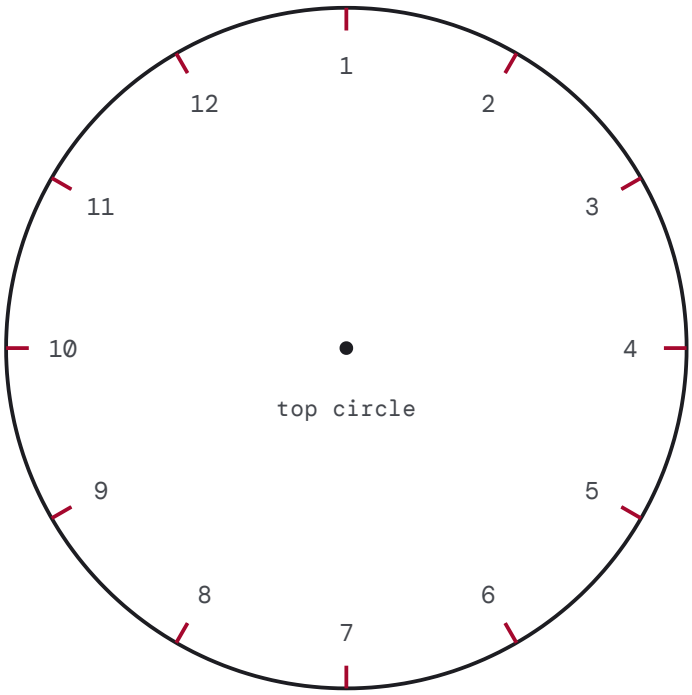


join each number to its triple

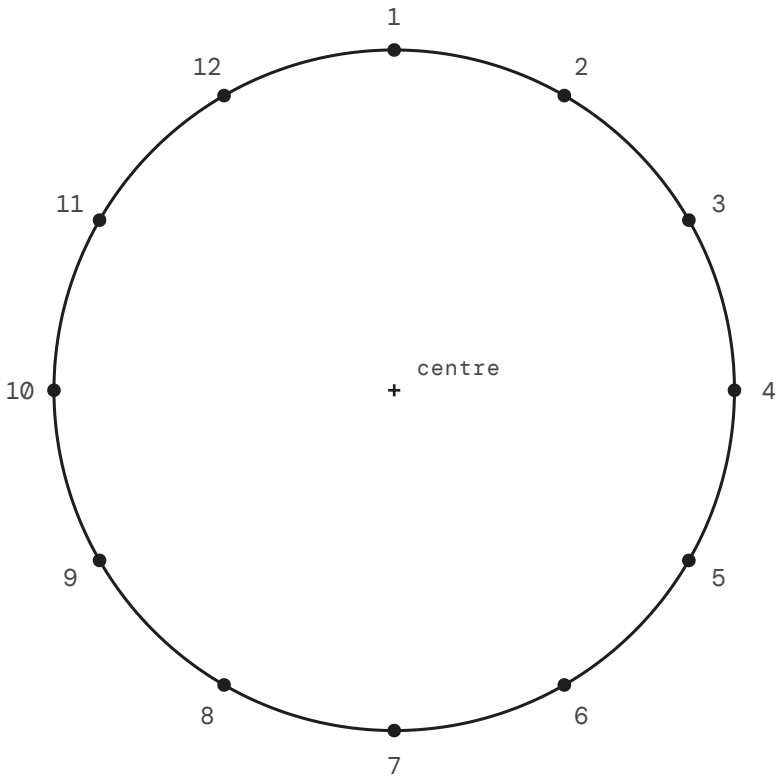
Template for part B. Print at 100% and stick it onto card.

Cut out both circles and notch each mark





For drawing: the waist seen from above (don't cut this one out)



Curves made of straight lines

SOLUTIONS AND NOTES

AIMS

Discover that a curve can emerge from a family of straight lines, find patterns with multiplication and remainders, and compare a measurement with a geometric construction.

MATHS TOPICS

Geometry · Measurement · Patterns.

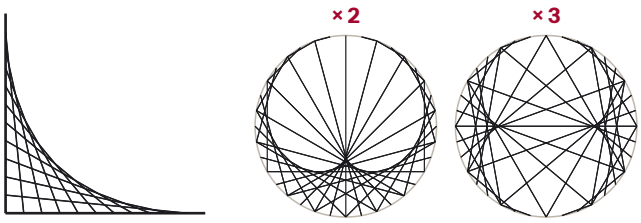
TIPS

- In part A, a sharp pencil and a clear ruler give much cleaner curves.
- Elastic thread keeps all twelve threads taut. Without it, use ordinary thread and pull the top circle upwards so the threads stay taut. As you turn it, it will drop a little, but the waist doesn't change.
- To measure the waist, hold the ruler level at mid-height without pressing on the threads.

GAUDÍ AND RULED SURFACES

Gaudí used hyperboloids for the skylights of the Sagrada Família, hyperbolic paraboloids in vaults and joints, and conoids for the roof of the Schools. All of them are ruled surfaces: through every point runs at least one whole straight line.

SOLUTIONS



the angle · "× 2", a cardioid · "× 3", a nephroid

- A1 A curve that rounds off the corner: an arc of a parabola, the curve that touches every line.
- A2 Smoother: there are more lines, closer together. The curve stays in the same place.
- A3 A heart with one tip. Cardioid comes from the Greek kardia, heart.
- A4 × 3: two tips (a nephroid, "kidney-shaped"). × 2: one. × 4: three. Multiplying by k gives $k - 1$ tips.

Notches	0	1	2	3	4	5	6
Waist (cm)	9.0	8.7	7.8	6.4	4.5	2.3	0

- B2 Yes, within a few millimetres. The threads sit in the notches, 3 mm in from the edge, so the measured waist comes out a little narrower: 8.4 cm with no turn. At 180° all the threads pass through the centre: the waist vanishes and you get two cones meeting at their tips.
- B3 Seen from above, each thread is a chord of the circle. The point of the thread closest to the axis is its middle, at mid-height, and its distance from the centre is that of the chord's midpoint. For older students: waist = $D \cdot \cos(\text{turn} / 2)$.

To follow up: chapter 04 at crm.cat/les-matematiques-de-gaudi, with a hyperboloid you can twist; Red Blob Games' "Mathologer's multiplication on a circle with modulo", which draws times tables on a circle; and the video "Voltes foradades" (secondary, in Catalan) at educa.sagradafamilia.org.

FURTHER READING

- Alsina, C. and Gómez Serrano, J. (2002). "Gaudí, geométricamente". *La Gaceta de la RSME*, 5(3), 523–539. In Spanish.
- Alsina, C. (2016). "Un templo de Geometría: la Sagrada Familia en 2026". *Suma*, 82, 35–42. In Spanish, free to read on the journal's website.

The full bibliography, with links, is at crm.cat/les-matematiques-de-gaudi.