ACTIVITY 1 · CHAPTER 01 · THE CATENARY

Hang a chain

Galileo wrote that a hanging chain traces a parabola, and that the match is better the less it sags. With a small chain, two drawing pins and a ruler, you can check.



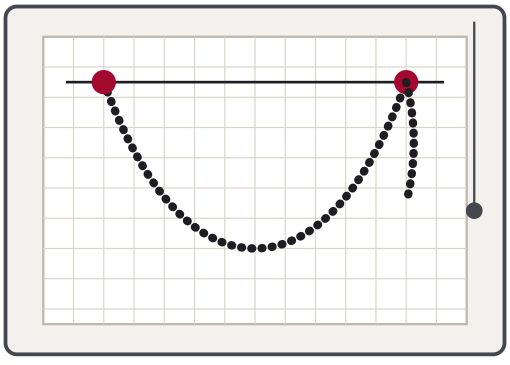
Name _____ Class _____ Date _____

TIME	50 minutes
GROUPS	Groups of three
YOU NEED	A ball chain about 50 cm long (the kind used for blinds or key rings), two drawing pins, a cork board or a piece of thick cardboard, the grid on page 3, a ruler, a pencil and three colours.

Predict Before you hang the chain: what shape do you think it will make? Sketch it freehand between the two dots.

1 Hang the chain

1. Print the grid without resizing it and check with the ruler that the control bar is 5 cm long.
2. Stick the grid to the board and stand it up against the wall. The line from A to B must be horizontal.
3. Push one pin in at A and another at B and hang the chain from them. Lengthen or shorten it until its lowest point touches V_1, 5 cm below. Let the spare chain hang down beyond the pins.
4. When it is still, make a coloured dot wherever the chain crosses one of the marked vertical lines. Take the chain off and join the dots.
5. Hang it again, longer this time, until it touches V_2, 15 cm below, and repeat step 4 in another colour.



grid on the board, pins and plumb line

A paper clip hanging from a thread shows you the vertical. Gravity is handy for that too.

2 Work out the parabola

You can work out, point by point, the parabola that goes through A, B and the lowest point. If x is the distance from the centre, in centimetres, the depth below the line AB is:

5 cm parabola: depth = $5 - x \times x \div 20$

15 cm parabola: depth = $15 - 0.15 \times x \times x$

x (cm)	0	2	4	6	8	10
5 cm parabola	5					0
15 cm parabola	15					0

> The curve is symmetrical: at $x = -4$ the depth is the same as at $x = 4$. Plot the points from the table on both sides of the centre in your third colour and join them.

3 Compare

Q1 With the 5 cm chain, what is the largest gap between the chain and the parabola? _____ mm

Q2 And with the 15 cm chain? _____ mm. Where is the gap largest: near the centre, near the pins or halfway?

Q3 Does the chain hang above or below the parabola?

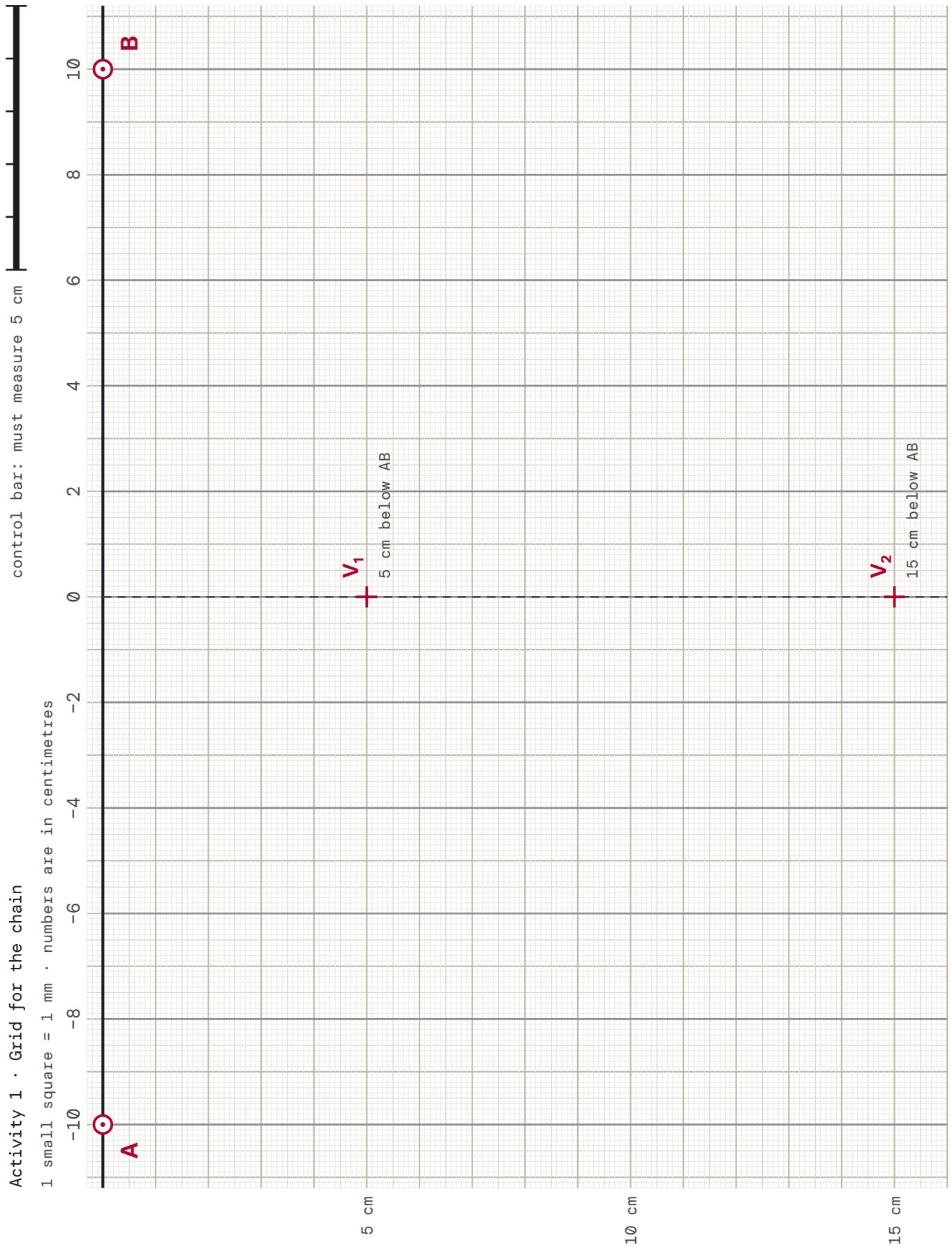
Q4 Galileo said the match gets better the less the chain sags. Do your measurements agree? Use two of your numbers to explain.

Q5 Turn the sheet upside down: the chain has become an arch. Draw arrows at three points of the arch showing which way each piece of the arch pushes on the piece below it. Gothic vaults needed buttresses, supporting walls outside the building, and Gaudí called them “crutches”. Why do you think his arch doesn’t need any?

GOING FURTHER

Look for catenaries outside the classroom: a cable between two poles, a washing line, a chain slung between two posts. In the attic of La Pedrera, in Barcelona, there are 270 of them, made of brick and upside down: they are the arches that hold up the roof.

Give the sheet a quarter turn clockwise so that the line AB is at the top. Before you start, check with a ruler that the control bar is 5 cm long.



Hang a chain

SOLUTIONS AND NOTES

AIMS

Calculate and plot points from an expression, work with decimals, compare a measurement with a model and explain why a shape is good for building.

MATHS TOPICS

Algebra (tables and graphs from an expression) · Number (decimals) · Measurement.

TIPS

- Ball chain is sold in hardware shops for blinds and can be cut with pliers. A thin necklace also works.
- Check the control bar: if the printer has shrunk the page, the measurements won't match the table.
- Without a cork board, thick cardboard taped to the wall, with the chain taped at A and B, does the job.
- Keep the board vertical. If it leans, the chain rubs on the paper and doesn't hang freely.

WHY THE CHAIN ISN'T A PARABOLA

A parabola is the shape of a cable carrying a load spread evenly along the horizontal, like the deck of a suspension bridge. The chain carries the same weight for every centimetre of chain, and near the pins, where it is steeper, there is more chain for every centimetre of width. The exact curve is the catenary, $y = a \cdot \cosh(x / a)$, found in 1691 by Leibniz, Huygens and Johann Bernoulli, independently.

SOLUTIONS

x (cm)	0	2	4	6	8	10
Parabola 5	5	4.8	4.2	3.2	1.8	0
Chain 5*	5	4.8	4.2	3.3	1.9	0
Parabola 15	15	14.4	12.6	9.6	5.4	0
Chain 15*	15	14.6	13.2	10.7	6.5	0

* Values worked out from the catenary. Real measurements may be 1 or 2 mm off at each point.

- Q1 Under 1 mm (0.9 mm at most, around $x = \pm 7$). In practice they coincide.
- Q2 About 12 mm, between the centre and the pins, closer to the pins (around $x = \pm 7$). On the marked lines students will see about 11 mm, at $x = \pm 8$.
- Q3 Below: the chain hangs lower than the parabola.
- Q4 Yes: under 1 mm when the chain sags a little, over 1 cm when it sags a lot.
- Q5 The arrows run along the arch, tangent to the curve. The push follows the shape of the arch down to the ground: none of it escapes from the stone, so nothing outside has to catch it. In a Gothic vault the push arrives at the top of the walls and has to be carried down by buttresses and flying buttresses.

To follow up: chapter 01 at crm.cat/les-matematiques-de-gaudi, with a chain you can turn over, and the video "Catenàries per superar el gòtic" (secondary, in Catalan) at educa.sagradafamilia.org.

FURTHER READING

- Block, P., DeJong, M. and Ochsendorf, J. (2006). "As Hangs the Flexible Line: Equilibrium of Masonry Arches". *Nexus Network Journal*, 8(2), 13–24. Free to read on the MIT website.
- Fernández Jiménez, A. (2020). "La catenaria y su influencia en la arquitectura de Gaudí". *La Gaceta de la RSME*, 23(2), 303–323. In Spanish.

The full bibliography, with links, is at crm.cat/les-matematiques-de-gaudi.