ACTIVITY BOOKLET · FOR TEACHERS AND STUDENTS

The mathematics of Gaudí in the classroom



Six activities for students aged 12 to 14 that between them cover the eight chapters of "The mathematics of Gaudí", a travelling exhibition by the Centre de Recerca Matemàtica that opened at the UAB Science and Technology Library in October 2026. They all use classroom materials: thread, paper clips, tracing paper, a small chain, a cardboard tube. The idea is for students to get their hands on the geometry before they put a name to it.

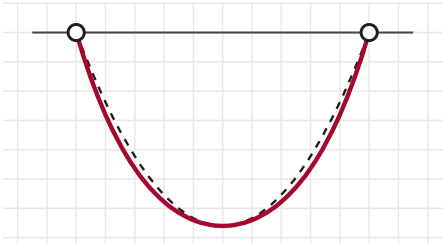
Each activity has student worksheets, templates to print where needed and a page of solutions and notes for teachers. They can be done in any order, and nobody needs to have seen the exhibition. Each one points to the matching chapter of the web page, where the figures move.

	Activity	Maths topics	Time	Main materials
1	**Hang a chain** Chapter 01	Algebra, number, measurement	50 min	Ball chain, two drawing pins, cork board
2	**An upside-down church** Chapters 02–03	Geometry, measurement	25 + 50 min	Thread, paper clips, shoebox lid, phone
3	**Curves made of straight lines** Chapter 04	Geometry, measurement, patterns	30 + 50 min	Ruler, card, elastic thread
4	**A straight line that coils** Chapter 05	Measurement, number, geometry	50 min	Cardboard tube, sticky tape
5	**The climbing star** Chapters 06–07	Geometry, number, proportion	50 min	Tracing paper, drawing pin, eraser
6	**Your own (almost) magic square** Chapter 08	Number, algebra	50 min	Pencil and colours

Print the templates at 100%, without fit-to-page. Every sheet works in black and white. Each activity can also be downloaded on its own at crm.cat/les-matematiques-de-gaudi.

ACTIVITY 1 · CHAPTER 01 · THE CATENARY

Hang a chain



Galileo wrote that a hanging chain traces a parabola, and that the match is better the less it sags. With a small chain, two drawing pins and a ruler, you can check.

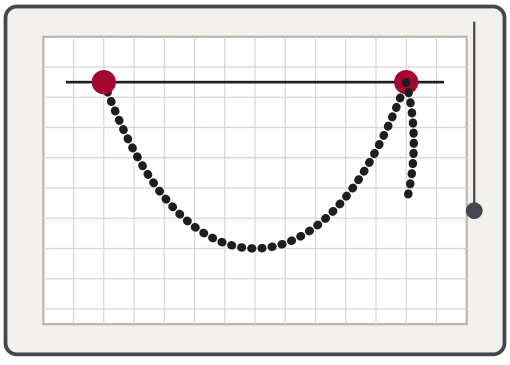
Name _____ Class _____ Date _____

TIME	50 minutes
GROUPS	Groups of three
YOU NEED	A ball chain about 50 cm long (the kind used for blinds or key rings), two drawing pins, a cork board or a piece of thick cardboard, the grid on page 4, a ruler, a pencil and three colours.

Predict Before you hang the chain: what shape do you think it will make? Sketch it freehand between the two dots.

1 Hang the chain

1. Print the grid without resizing it and check with the ruler that the control bar is 5 cm long.
2. Stick the grid to the board and stand it up against the wall. The line from A to B must be horizontal.
3. Push one pin in at A and another at B and hang the chain from them. Lengthen or shorten it until its lowest point touches V_1, 5 cm below. Let the spare chain hang down beyond the pins.
4. When it is still, make a coloured dot wherever the chain crosses one of the marked vertical lines. Take the chain off and join the dots.
5. Hang it again, longer this time, until it touches V_2, 15 cm below, and repeat step 4 in another colour.



grid on the board, pins and plumb line

A paper clip hanging from a thread shows you the vertical. Gravity is handy for that too.

2 Work out the parabola

You can work out, point by point, the parabola that goes through A, B and the lowest point. If x is the distance from the centre, in centimetres, the depth below the line AB is:

5 cm parabola: depth = $5 - x \times x \div 20$

15 cm parabola: depth = $15 - 0.15 \times x \times x$

x (cm)	0	2	4	6	8	10
5 cm parabola	5					0
15 cm parabola	15					0

> The curve is symmetrical: at $x = -4$ the depth is the same as at $x = 4$. Plot the points from the table on both sides of the centre in your third colour and join them.

3 Compare

Q1 With the 5 cm chain, what is the largest gap between the chain and the parabola? _____ mm

Q2 And with the 15 cm chain? _____ mm. Where is the gap largest: near the centre, near the pins or halfway?

Q3 Does the chain hang above or below the parabola?

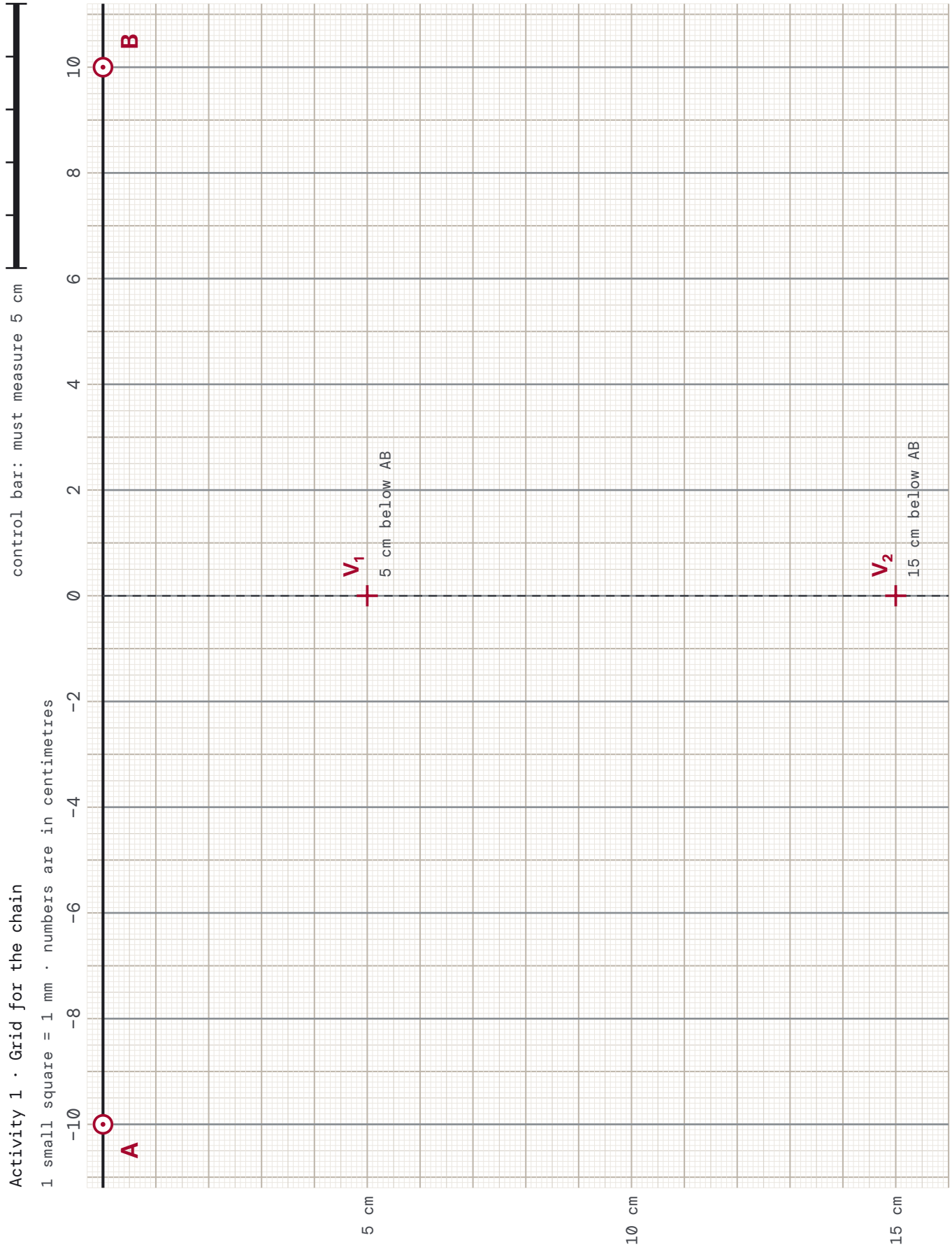
Q4 Galileo said the match gets better the less the chain sags. Do your measurements agree? Use two of your numbers to explain.

Q5 Turn the sheet upside down: the chain has become an arch. Draw arrows at three points of the arch showing which way each piece of the arch pushes on the piece below it. Gothic vaults needed buttresses, supporting walls outside the building, and Gaudí called them “crutches”. Why do you think his arch doesn’t need any?

GOING FURTHER

Look for catenaries outside the classroom: a cable between two poles, a washing line, a chain slung between two posts. In the attic of La Pedrera, in Barcelona, there are 270 of them, made of brick and upside down: they are the arches that hold up the roof.

Give the sheet a quarter turn clockwise so that the line AB is at the top. Before you start, check with a ruler that the control bar is 5 cm long.



Hang a chain

SOLUTIONS AND NOTES

AIMS

Calculate and plot points from an expression, work with decimals, compare a measurement with a model and explain why a shape is good for building.

MATHS TOPICS

Algebra (tables and graphs from an expression) · Number (decimals) · Measurement.

TIPS

- Ball chain is sold in hardware shops for blinds and can be cut with pliers. A thin necklace also works.
- Check the control bar: if the printer has shrunk the page, the measurements won't match the table.
- Without a cork board, thick cardboard taped to the wall, with the chain taped at A and B, does the job.
- Keep the board vertical. If it leans, the chain rubs on the paper and doesn't hang freely.

WHY THE CHAIN ISN'T A PARABOLA

A parabola is the shape of a cable carrying a load spread evenly along the horizontal, like the deck of a suspension bridge. The chain carries the same weight for every centimetre of chain, and near the pins, where it is steeper, there is more chain for every centimetre of width. The exact curve is the catenary, $y = a \cdot \cosh(x / a)$, found in 1691 by Leibniz, Huygens and Johann Bernoulli, independently.

SOLUTIONS

x (cm)	0	2	4	6	8	10
Parabola 5	5	4.8	4.2	3.2	1.8	0
Chain 5*	5	4.8	4.2	3.3	1.9	0
Parabola 15	15	14.4	12.6	9.6	5.4	0
Chain 15*	15	14.6	13.2	10.7	6.5	0

* Values worked out from the catenary. Real measurements may be 1 or 2 mm off at each point.

- Q1 Under 1 mm (0.9 mm at most, around $x = \pm 7$). In practice they coincide.
- Q2 About 12 mm, between the centre and the pins, closer to the pins (around $x = \pm 7$). On the marked lines students will see about 11 mm, at $x = \pm 8$.
- Q3 Below: the chain hangs lower than the parabola.
- Q4 Yes: under 1 mm when the chain sags a little, over 1 cm when it sags a lot.
- Q5 The arrows run along the arch, tangent to the curve. The push follows the shape of the arch down to the ground: none of it escapes from the stone, so nothing outside has to catch it. In a Gothic vault the push arrives at the top of the walls and has to be carried down by buttresses and flying buttresses.

To follow up: chapter 01 at crm.cat/les-matematiques-de-gaudi, with a chain you can turn over, and the video "Catenàries per superar el gòtic" (secondary, in Catalan) at educa.sagradafamilia.org.

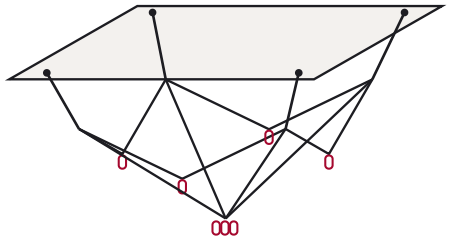
FURTHER READING

- Block, P., DeJong, M. and Ochsendorf, J. (2006). "As Hangs the Flexible Line: Equilibrium of Masonry Arches". *Nexus Network Journal*, 8(2), 13–24. Free to read on the MIT website.
- Fernández Jiménez, A. (2020). "La catenaria y su influencia en la arquitectura de Gaudí". *La Gaceta de la RSME*, 23(2), 303–323. In Spanish.

The full bibliography, with links, is at crm.cat/les-matematiques-de-gaudi.

ACTIVITY 2 · CHAPTERS 02 AND 03 · THE HANGING MODEL

An upside-down church



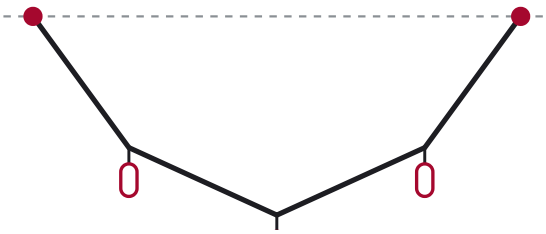
To design the church at the Colònia Güell, Gaudí hung a model made of strings and little bags of lead shot from the ceiling, and worked on it for about ten years. He photographed it, turned the photo upside down and drew the building on top. You will make a small one with thread and paper clips.

Name _____ Class _____ Date _____

TIME	Part A, 25 minutes. Part B, a 50-minute lesson.
GROUPS	Groups of three or four
YOU NEED	Thick sewing thread or thin string (about 3 m), around thirty small paper clips, a ruler, a protractor, an A3 sheet (or two A4 sheets taped together), a shoebox lid, sticky tape, scissors, two piles of books about 30 cm high, a phone or tablet to take photos and the template on page 8. To make the holes, an awl, to be used by an adult.

A One thread and three clips

1. Tie a thread between two points at the same height and 30 cm apart, so that 40 cm of thread hangs between them: two pins in a cork board, or two equal piles of books.
2. Bend three paper clips into hooks and hang them on the thread at 10, 20 and 30 cm from one end.
3. Hold the A3 sheet behind the thread and mark where the clips and the two ends are. Join the marks with the ruler.



40 cm thread with three equal clips

A1 Between two clips, does the thread curve or run straight? Where does it change direction?

A2 Measure with a protractor the angle the thread makes at each clip. Then hang a second clip next to the first and measure again.

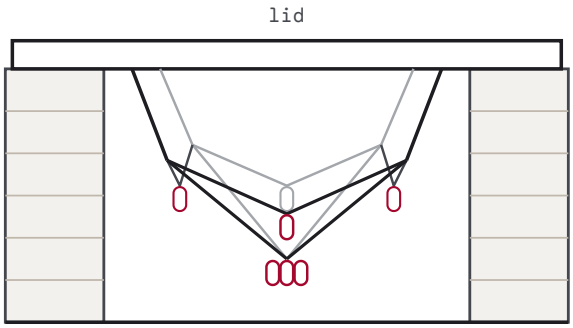
	Clip 1	Clip 2	Clip 3
One clip at each point			
Two clips at the first point			

A3 What happens to the angle when there is more weight? And to the rest of the thread?

A4 Turn the drawing upside down and imagine it made of stone, with the weights on top: each section pushes on the next and the whole thing stands up. Engineers call it a funicular polygon. If you wanted to put more weight at one point, what would the angle there have to be like?

B The church in a box

1. Cut out the template on page 8, leaving a 1 cm margin around the square, and stick it in the middle of the outside of the lid. An adult makes a hole at each point C.
2. Cut the threads: four columns of 10 cm, four arches of 24 cm and two diagonals of 32 cm. These lengths include the knots.
3. Push each column through a hole and tie a big knot on top. Tie a paper clip to the other end: it will be a joint of the net.
4. Tie each arch between two neighbouring clips and each diagonal between two opposite clips. Where the diagonals cross, tie them together.
5. Put the lid on the piles of books with the threads hanging down. Hang one clip from the middle of each arch and three from the centre.



the lid on two piles of books, with the net hanging underneath

B1 Look at the column threads. Are they vertical? Which way do they lean?

B2 Hang three more clips from the centre. What do the columns do? And the arches?

B3 Take a photo from the side, with the phone level with the net, and rotate it 180°. Where are the arches, the columns and the vault?

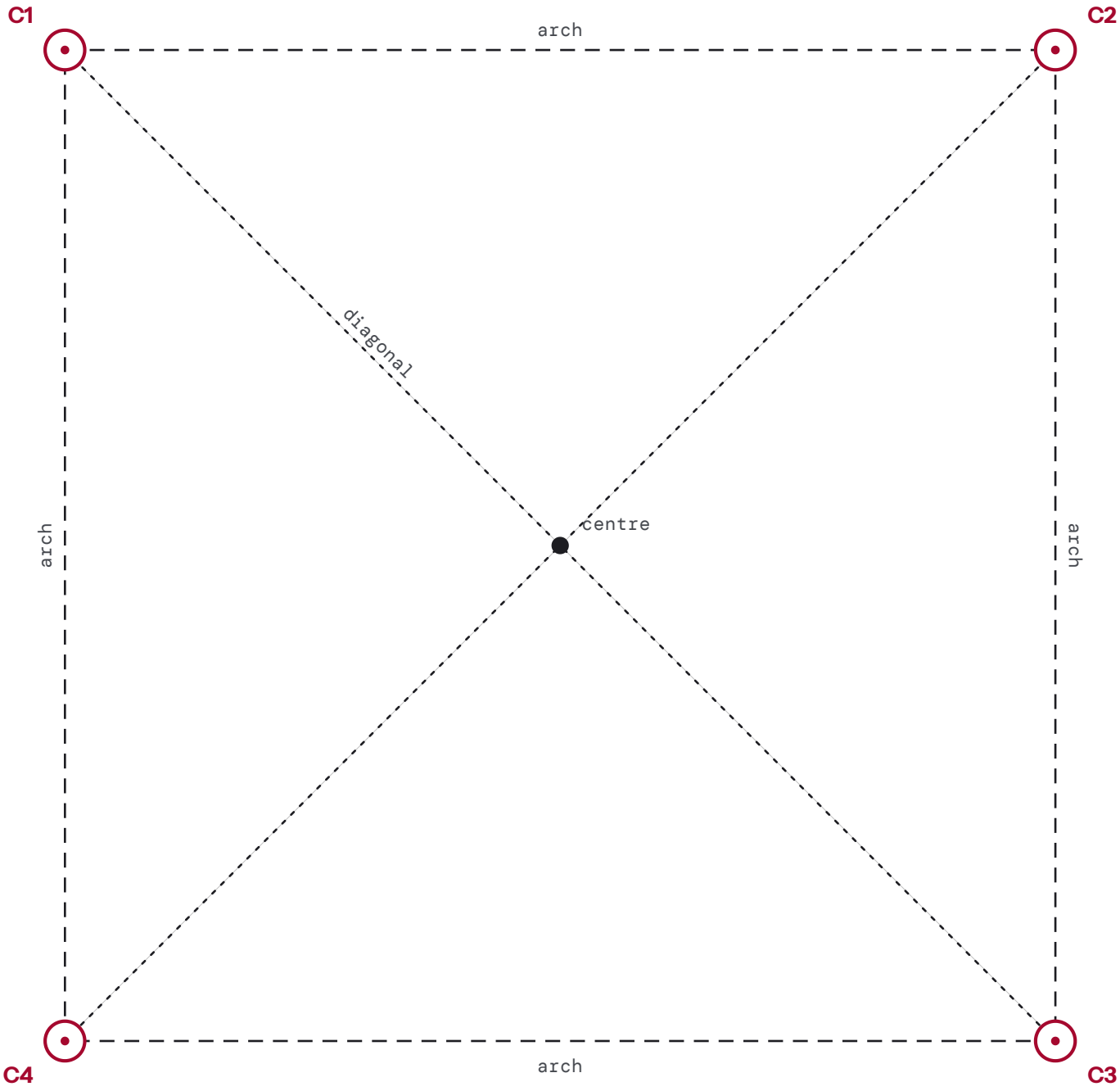
B4 Print the rotated photo, or trace it onto tracing paper held against the screen, and draw walls, windows and a door on it. Gaudí did exactly this on the upside-down photos of his model. The Museu Nacional d'Art de Catalunya keeps two of them.

GOING FURTHER

Move a single weight and watch what moves: everything does. That is why Gaudí's model worked like a computer. When he changed one load, gravity recalculated the whole church in a few seconds.

Cut out the template with a 1 cm margin around the square. Holes only with an adult.

Floor plan of your church · stick it on the outside of the lid



C1-C4: holes for the columns · dotted lines: arches and diagonals

check: should measure 4 cm

Thread	How many	Length with knots	Goes from
Column	4	10 cm	a hole C to a clip
Arch	4	24 cm	a clip to the next clip
Diagonal	2	32 cm	a clip to the opposite clip

An upside-down church

SOLUTIONS AND NOTES

AIMS

See that a loaded thread forms straight sections, relate weight to angle, see that in a net everything depends on everything else, measure angles and make 180° turns.

MATHS TOPICS

Geometry (rotations) · Measurement (angles). Links with Design and Technology: structures, tension and compression.

TIPS

- A thread that doesn't stretch, such as thick cotton, gives cleaner shapes. Use identical clips so they weigh the same.
- To save time, you can stick the templates on and make the holes before part B. The lid needs to be at least 18 cm wide; if it is smaller or bends, a piece of thick cardboard does the same job. If the centre clips touch the table, add a book to each pile.
- For the photo, rest the phone on a pile of books, level with the threads. Any photo editor will rotate the image by 180°.
- Part A can use the cork board from activity 1.

GAUDÍ'S MODEL

Gaudí received the commission in 1898 and worked with the model for about ten years, at 1:10 scale and loaded with little bags of lead shot. Building began in 1908 and stopped in 1914, and only the crypt was built. Two of the upside-down photographs he drew on in charcoal and chalk, one of the outside and one of the inside, are in the Museu Nacional d'Art de Catalunya. In 1982–83 a team at Frei Otto's Institut für leichte Flächentragwerke in Stuttgart rebuilt the model, which is now in the Sagrada Família museum. The exhibition in the old cooperative building at the Colònia Güell has a reproduction.

SOLUTIONS

- A1 Between clips the thread runs straight. It changes direction at each clip.
- A2 With equal clips, calculation gives about 151° at the outer clips and about 131° at the middle one. With two clips at the first point, about 136°, 139° and 154°. Real measurements can be a few degrees off.
- A3 The more weight, the sharper the angle. And the whole thread moves, not just that point.
- A4 Sharper, more pointed.
- B1 No. They lean towards the centre, because the arches and diagonals pull them inwards.
- B2 The columns lean a little further in and the arches are pulled towards the centre. The changes are only a few millimetres: compare two photos taken from the same spot.
- B3 In the rotated photo the lid is the floor. The columns rise leaning inwards, and the arches and diagonals become arches and a vault, highest at the centre.

To follow up: chapters 02 and 03 at crm.cat/les-matematiques-de-gaudi, with an interactive string and net; the MNAC online exhibition "Gaudí, the Architect", showing both drawn-on photographs; and the video "El mètode de models invertits. La Colònia Güell" (in Catalan) at educa.sagradafamilia.org.

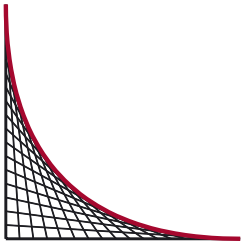
FURTHER READING

- Tomlow, J. (1989). *Das Modell / The Model / El modelo* (IL 34). Stuttgart: Institut für leichte Flächentragwerke. Free to read on the ILEK website.
- Huerta, S. (2006). "Structural Design in the Work of Gaudí". *Architectural Science Review*, 49(4), 324–339.
- Roca i Fabregat, P. (2005). *Antoni Gaudí i el disseny estructural*. Barcelona: Institut d'Estudis Catalans. In Catalan, free to read on the IEC website.

The full bibliography, with links, is at crm.cat/les-matematiques-de-gaudi.

ACTIVITY 3 · CHAPTER 04 · RULED SURFACES

Curves made of straight lines



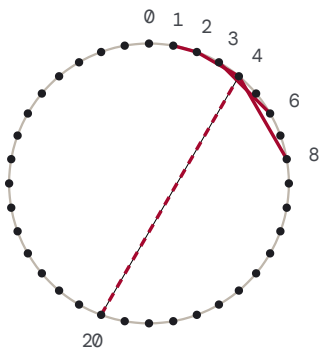
In this activity you will only draw straight lines, with a ruler. The curves will appear on their own. Gaudí used this to build curved shapes out of straight pieces, like the skylights in the vaults of the Sagrada Família.

Name _____ Class _____ Date _____

TIME	Part A, about 30 minutes. Part B, a 50-minute lesson.
GROUPS	Part A, on your own. Part B, in pairs.
YOU NEED	Ruler, coloured pencils, the templates on pages 12 and 13, card, glue, scissors and about 2 m of elastic thread (the kind used for bracelets).

A Straight lines that make curves

1. On the template on page 12 there is a right angle with its sides numbered 1 to 10. With the ruler, join the 1 on one side to the 1 on the other, 2 to 2, and so on up to 10. The first line is already drawn.
2. On the "× 2" circle there are 36 points numbered 0 to 35. Join each number to its double: 1 to 2, 2 to 4, 3 to 6… When the double goes past 35, subtract 36: 20 goes with 40 − 36 = 4.
3. On the "× 3" circle, do the same with the triple: 1 to 3, 2 to 6… Subtract 36 as many times as you need.



how "× 2" starts: 1 to 2, 2 to 4, 3 to 6, 4 to 8… and 20 to 4

A1 In the angle you have drawn ten straight lines. What shape appears?

A2 If each side had 20 marks instead of 10, would the curve look smoother or rougher? Why?

A3 What figure appears on the "× 2" circle? It is called a cardioid. Where do you think the name comes from?

A4 How many tips does the "× 3" figure have? And the "× 2" one? Guess how many tips the "× 4" figure would have.

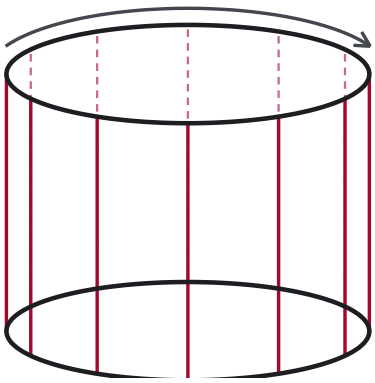
B A thread hyperboloid

1 Stick page 13 onto card and cut out the bottom circle and the top circle. Make a small notch, 3 mm deep, at each mark.

2 Tie the elastic thread to notch 1 on the bottom circle. Take it up to notch 1 on the top circle, over to notch 2, down to notch 2 on the bottom circle, and so on up to 12. Tie off the end.

3 A partner holds the bottom circle on the table. You hold the top one about 12 cm higher, notch above notch. You have a cylinder made of straight threads.

4 Turn the top circle by one notch, two, three... up to six. Each time, measure the width of the waist at its narrowest point with the ruler, and write it in the table.



the thread goes up and down through the notches

Notches turned	0	1	2	3	4	5	6
Degrees	0	30	60	90	120	150	180
Waist measured (cm)							
Waist from drawing (cm)							

B1 Check by drawing. On the circle you don't cut out, on page 13, draw the line from point 1 to point 4: it is what one thread looks like from above when you turn three notches. Mark its midpoint, measure its distance from the centre and multiply by 2. Do it for each turn (1 to 2, 1 to 3...) and fill in the last row of the table.

B2 Do the last two rows of the table match? What happens when you turn six notches (180°)?

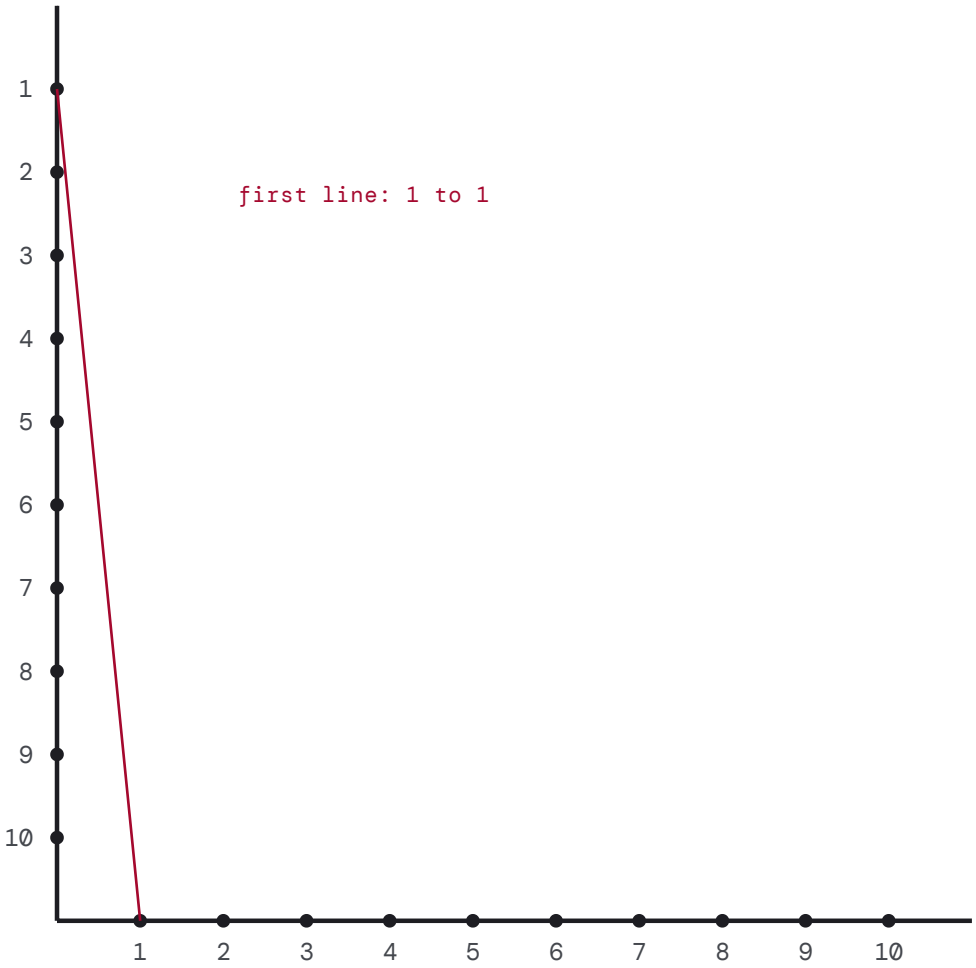
B3 Why do the drawing and the model give the same result? Hint: look at the model from above.

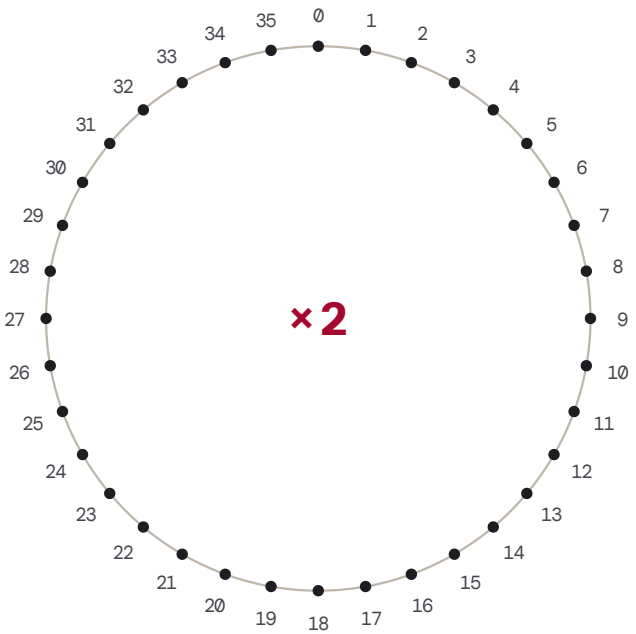
GOING FURTHER

Try "× 4", "× 5" and "× 6" on circles you draw yourself and check whether your guess in question A4 works. And look at your hyperboloid: it is all curves, yet it is made only of straight threads. That is why it can be built with bars, formwork planks or a taut thread.

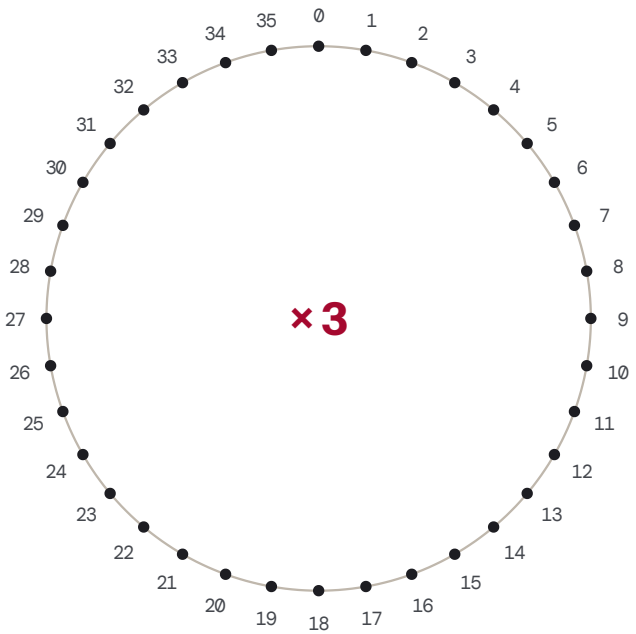
Template for part A. Nothing to cut out.

Straight lines that make a curve





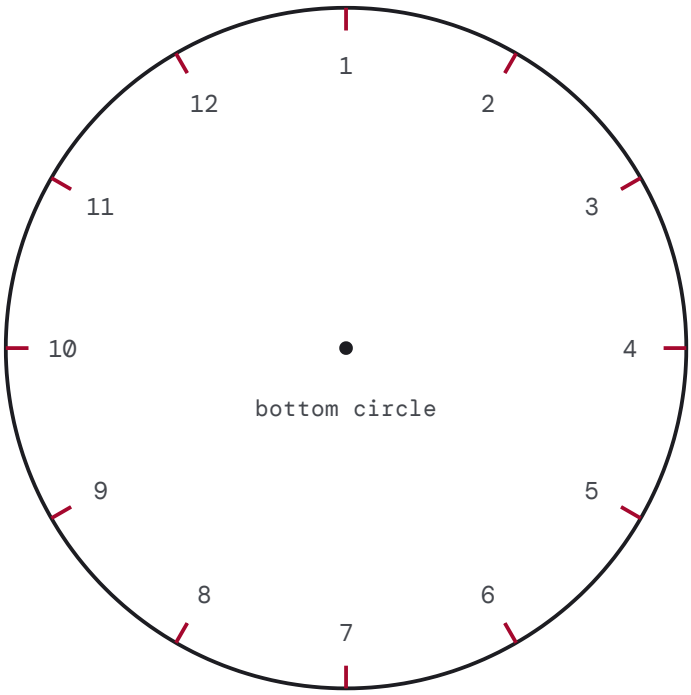
join each number to its double

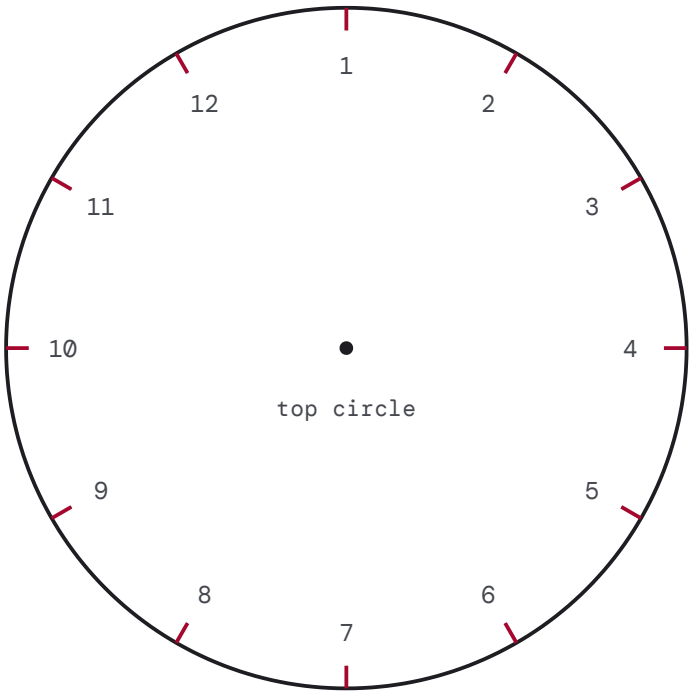


join each number to its triple

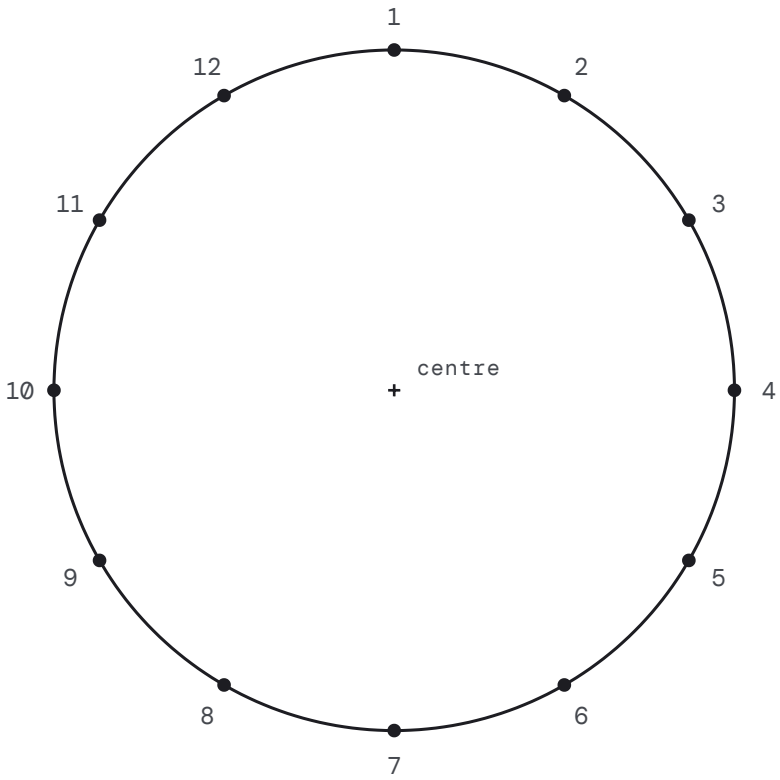
Template for part B. Print at 100% and stick it onto card.

Cut out both circles and notch each mark





For drawing: the waist seen from above (don't cut this one out)



Curves made of straight lines

SOLUTIONS AND NOTES

AIMS

Discover that a curve can emerge from a family of straight lines, find patterns with multiplication and remainders, and compare a measurement with a geometric construction.

MATHS TOPICS

Geometry · Measurement · Patterns.

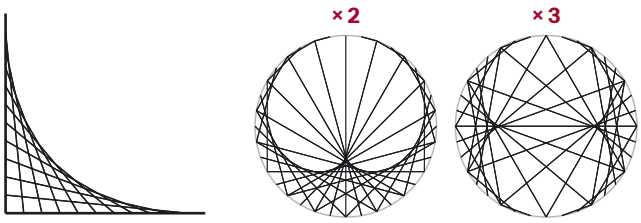
TIPS

- In part A, a sharp pencil and a clear ruler give much cleaner curves.
- Elastic thread keeps all twelve threads taut. Without it, use ordinary thread and pull the top circle upwards so the threads stay taut. As you turn it, it will drop a little, but the waist doesn't change.
- To measure the waist, hold the ruler level at mid-height without pressing on the threads.

GAUDÍ AND RULED SURFACES

Gaudí used hyperboloids for the skylights of the Sagrada Família, hyperbolic paraboloids in vaults and joints, and conoids for the roof of the Schools. All of them are ruled surfaces: through every point runs at least one whole straight line.

SOLUTIONS



the angle · "× 2", a cardioid · "× 3", a nephroid

- A1 A curve that rounds off the corner: an arc of a parabola, the curve that touches every line.
- A2 Smoother: there are more lines, closer together. The curve stays in the same place.
- A3 A heart with one tip. Cardioid comes from the Greek kardia, heart.
- A4 × 3: two tips (a nephroid, "kidney-shaped"). × 2: one. × 4: three. Multiplying by k gives $k - 1$ tips.

Notches	0	1	2	3	4	5	6
Waist (cm)	9.0	8.7	7.8	6.4	4.5	2.3	0

- B2 Yes, within a few millimetres. The threads sit in the notches, 3 mm in from the edge, so the measured waist comes out a little narrower: 8.4 cm with no turn. At 180° all the threads pass through the centre: the waist vanishes and you get two cones meeting at their tips.
- B3 Seen from above, each thread is a chord of the circle. The point of the thread closest to the axis is its middle, at mid-height, and its distance from the centre is that of the chord's midpoint. For older students: waist = $D \cdot \cos(\text{turn} / 2)$.

To follow up: chapter 04 at crm.cat/les-matematiques-de-gaudi, with a hyperboloid you can twist; Red Blob Games' "Mathologer's multiplication on a circle with modulo", which draws times tables on a circle; and the video "Voltes foradades" (secondary, in Catalan) at educa.sagradafamilia.org.

FURTHER READING

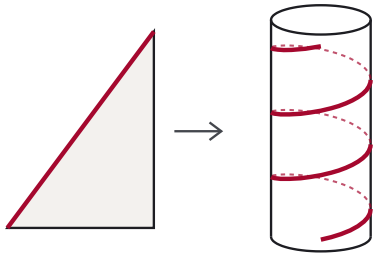
- Alsina, C. and Gómez Serrano, J. (2002). "Gaudí, geométricamente". *La Gaceta de la RSME*, 5(3), 523–539. In Spanish.
- Alsina, C. (2016). "Un templo de Geometría: la Sagrada Familia en 2026". *Suma*, 82, 35–42. In Spanish, free to read on the journal's website.

The full bibliography, with links, is at crm.cat/les-matematiques-de-gaudi.

ACTIVITY 4 · CHAPTER 05 · LIVING CURVES

A straight line that coils

Cut out a paper triangle and roll it around a tube: its slanted edge turns into a helix, the curve of springs and of spiral-staircase handrails.

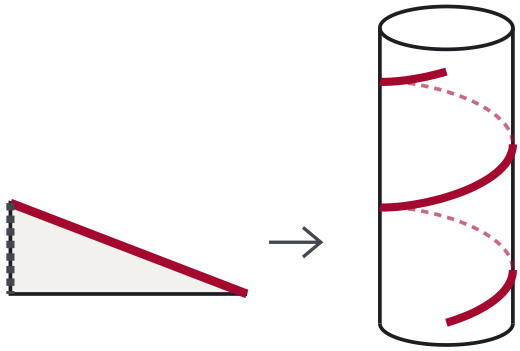


Name _____ Class _____ Date _____

TIME	50 minutes
GROUPS	In pairs
YOU NEED	The template on page 17, a cardboard tube (from kitchen roll or toilet roll), scissors, sticky tape and a ruler.

A From triangle to helix

1. Cut out triangle 1 and the measuring strip. The long slanted edge of the triangle is already coloured.
2. Tape the short side of the triangle (the 5 cm one) along the tube, from top to bottom rather than around it.
3. Roll the triangle tightly around the tube and tape down the tip.



the triangle rolls up and its edge makes a helix

A1 Wrap the strip around the tube and measure its circumference: C = _____ cm

A2 How many times does the triangle go round the tube? Divide: 26 ÷ C = _____

A3 How far does the coloured line rise in each turn? Use a proportion: if it rises 5 cm over a width of 26 cm, over C cm it rises _____ cm. Now measure it on the tube. Does it match?

A4 Repeat with triangle 2, which is twice as tall. How far does it rise in each turn now? _____ cm

A5 Look at the tube from the side, with your eyes level with it, and draw the part of the line you can see. Now imagine the tube is made of glass and add the hidden part as a dashed line. What shape do the two parts make together?

A6 Photographed from above, a spiral staircase looks like a flat spiral, like a sea snail's shell (there is a photo in chapter 05 of the web page). If the staircase is a helix, why do the lower turns look smaller in the photo?

B An imaginary spiral staircase

Each step on this staircase rises 18 cm and turns 24°.

B1 How many steps make one full turn? (A full turn is 360°.) _____

B2 How much does the staircase rise in one turn? _____ cm

B3 A tower is 54 metres tall. How many steps do you climb to reach the top? How many turns does the staircase make?

B4 If you could unroll the staircase wall like the paper triangle, what shape would the handrail be? Draw the unrolled wall, with the handrail and a few steps.

HELIX OR HELICOID?

A helix is a line, like the edge of the triangle. A helicoid is a surface: the one swept by a straight line that turns and rises at the same time, like the ramp in a car park. Sort these: a spring, a car-park ramp, the handrail of a spiral staircase, a spiral slide, a DNA molecule.

Helix	
Helicoid	

tape this side to the tube

Triangle 2

26 × 10 cm

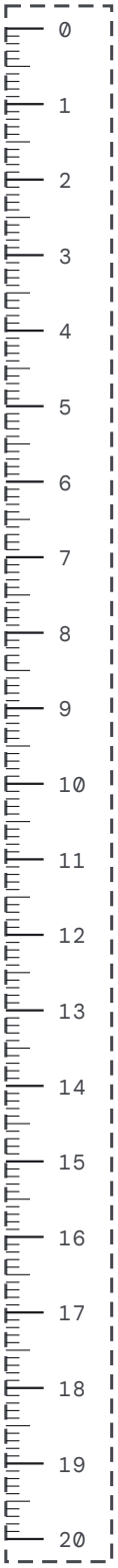
26 cm

26 cm

Triangle 1

26 × 5 cm

tape this side to the tube



measuring strip (cm)

Triangles to roll up and a measuring strip

Cut out both triangles and the strip. The thick edge is the one that will become a helix. The dotted side is the one you tape to the tube.

A straight line that coils

SOLUTIONS AND NOTES

AIMS

Connect a flat shape with the one it makes when rolled up, calculate with proportions, measure circumferences and use the degrees in a full turn.

MATHS TOPICS

Measurement · Number (proportion) · Geometry.

TIPS

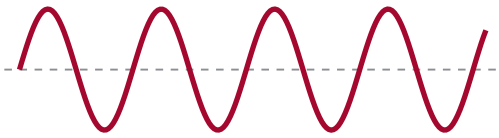
- Toilet-roll and kitchen-roll tubes are usually 12 to 14 cm round.
- If the triangle slips, start from the short side and roll the tube along the table.
- Part B needs no materials, only calculation.

GAUDÍ AND HELICES

Gaudí uses helices in the spiral staircases of the towers, in columns and in chimneys. The spiral you see when you photograph a spiral staircase from above is a perspective effect, not the logarithmic spiral of shells, even though it looks like one.

SOLUTIONS

- A1 Between 12 and 14 cm, depending on the tube.
- A2 26 ÷ C: about two turns.
- A3 5 × C ÷ 26. With C = 13 cm, 2.5 cm per turn.
- A4 Twice as much: with C = 13 cm, 5 cm per turn. The rise per turn is proportional to the height of the triangle.
- A5 A wave: a sine curve. Seen from the side, the helix $x = R\cos t, z = c\,t$ is the graph of a cosine. On the tube they only see the front half of each turn: slanted arcs from one edge to the other, one per turn.



- A6 Perspective: things further away look smaller.
- B1 360 ÷ 24 = 15 steps.
- B2 15 × 18 = 270 cm, that is, 2.70 m.
- B3 5,400 ÷ 18 = 300 steps, and 300 ÷ 15 = 20 turns.
- B4 A slanted straight line, like the edge of the triangle.
- Helix? Helix: the spring, the handrail and DNA (two helices, in fact). Helicoid: the ramp and the slide.

To follow up: chapter 05 at crm.cat/les-matematiques-de-gaudi, where the helix can be viewed from the side (a wave) or from above (a spiral).

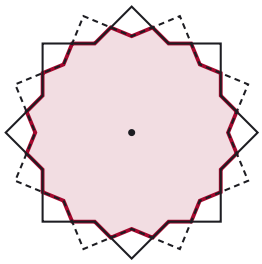
FURTHER READING

- Alsina, C., Gómez Serrano, J. and Faulí, J. (2004). "La geometria i la mecànica a l'obrador de Gaudí". *Revista de Tecnologia*, 1, 12–27. In Catalan.
- Giralt-Miracle, D. (ed.) (2002). *Gaudí. Exploring Form*. Barcelona: Lunwerg.

The full bibliography, with links, is at crm.cat/les-matematiques-de-gaudi.

ACTIVITY 5 · CHAPTERS 06 AND 07 · THE COLUMN AND THE MODULE

The climbing star



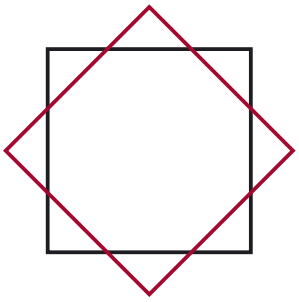
The columns of the Sagrada Família's central nave start as an eight-pointed star and become almost round as they rise. They do it by turning right and left at the same time. With two sheets of tracing paper you can see how.

Name _____ Class _____ Date _____

TIME	50 minutes
GROUPS	In pairs
YOU NEED	Two sheets of tracing paper (or the template on page 21 printed twice on tracing paper), a drawing pin, an eraser, coloured pencils, a ruler, a 16 cm strip of paper and scissors.

A Two turning stars

1. Trace the star and the ring of marks on page 21 onto two sheets of tracing paper.
2. Lay one sheet on the other, with the stars exactly on top of each other, and push the pin through the centre into the eraser.
3. Turn the top sheet until its mark 0 sits over mark 1 on the bottom sheet.
4. Colour in the part that is inside both stars at once.



two squares, one turned 45°

A1 How many points does the figure you coloured have? .....

A2 Neighbouring marks are 22.5° apart. Check: there are ..... marks in a full turn, and 360 ÷ ..... = ...

A3 Now turn the top sheet one more mark (45° in total). What happens to the common part?

A4 The column does something similar, but both stars turn at once, one to the right and the other to the left. The cross-section is the part they share, and at the top of the first section it already has sixteen points. If the process were repeated with the sixteen-pointed figure, how many points would you get? And the time after that? What happens to the shape when there are lots and lots of points?

B Sections that get shorter

In the eight-pointed column the first section is 8 metres tall. Each following section is half as tall as the one before and doubles the points again.

Section	Height of section	Total height	Points at the top
1	8 m	8 m	16
2			
3			
4			
5			

B1 If you could carry on the table forever, how high would the total get?

B2 In the real column the shaft ends after the third section, and the capital sits on top. How tall is the shaft, and how many points does it have at the top?

B3 Cut the 16 cm strip in half. Cut the piece you have left in half, and so on. Lay the cut pieces in a row: do they ever reach 16 cm?

B4 With fractions: 1/2 + 1/4 + 1/8 + 1/16 = _____. And if you add 1/32? _____ How much is left to reach 1? _____

C Columns you can count

The Sagrada Família has columns with 6, 8, 10 and 12 points. The number of points decides almost everything: the diameter is that many twelfths of 210 cm, and the first section is that many metres tall.

Points	Diameter	1st section	2nd section	3rd section	Shaft
6	105 cm	6 m	3 m	1.5 m	10.5 m
8		8 m			14 m
10					
12	210 cm				

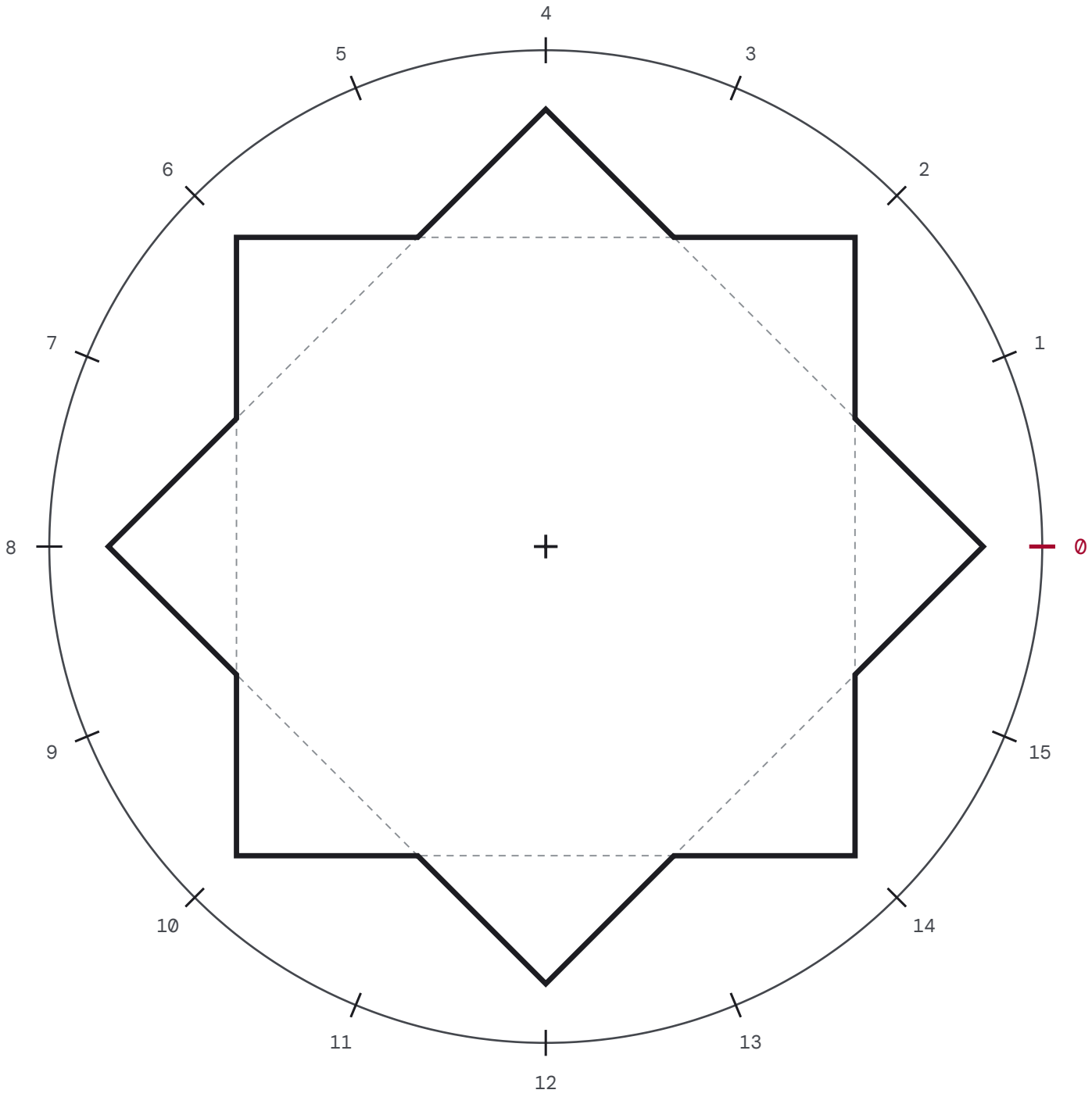
C1 How many centimetres of diameter go with each point? _____ cm

C2 Compare the diameter with the height of the shaft. How many times taller is it? _____

C3 The four twelve-pointed columns stand at the crossing, under the central tower, the tallest one. Why do you think they are the thickest?

Print this page twice on tracing paper, or trace it twice.

The eight-pointed star · trace it twice



marks every 22.5° · mark 0 sits on a point

The climbing star

SOLUTIONS AND NOTES

AIMS

Explore rotations and symmetry, discover an infinite sum with a finite result, work with fractions and recognise directly proportional quantities.

MATHS TOPICS

Geometry (rotation and symmetry) · Number (fractions) · Algebra (proportion).

TIPS

- Many laser printers take tracing paper. Otherwise trace the template or use acetate sheets.
- The eraser under the pin acts as an axle and protects the desk.
- In the real column the outline of the star is made of parabolic arcs rather than straight lines, but the process is the same.

WHY 22.5°

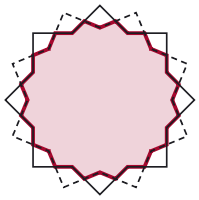
Any turn between 0° and 45° gives a common part with sixteen points, but only at 22.5°, half a step of the star, are they all equal. At 45° the two stars match again. So for the cross-section to have sixteen equal points, the two stars must end the first section 22.5° apart.

THE COLUMNS

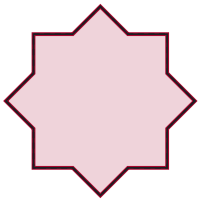
The twelve-pointed columns are the four at the crossing, in red porphyry; the ten-pointed ones, under the Evangelists' towers, are in black basalt.

To follow up: chapters 06 and 07 at crm.cat/les-matematiques-de-gaudi, where you can climb the column and watch its cross-section, and count the basilica's heights in 7.5 m modules; the Sagrada Família blog article "Sagrada Família columns: the geometry, mechanics and materials of a stone forest"; and the video "La columna de doble gir" (in Catalan) at educa.sagradafamilia.org.

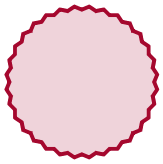
SOLUTIONS



22.5° turn: 16 equal points



45° turn: they match again



next section: 32 points

- A1 16.
- A2 16 marks; 360 ÷ 16 = 22.5°.
- A3 The stars match again: the common part is once more an eight-pointed star.
- A4 32, then 64… The shape gets closer and closer to a circle.

Section	2	3	4	5
Height	4 m	2 m	1 m	0.5 m
Total	12 m	14 m	15 m	15.5 m
Points	32	64	128	256

- B1 Up to 16 m, without ever reaching it.
- B2 14 m and 64 points. The remaining stretches are hidden inside the capital, an ellipsoid knot, and the whole column is 16 m tall, twice the number of points.
- B3 No: the piece you haven't cut is always missing.
- B4 15/16; 31/32; 1/32 is left.

Points	Diameter	Sections	Shaft
8	140 cm	8 + 4 + 2 m	14 m
10	175 cm	10 + 5 + 2.5 m	17.5 m
12	210 cm	12 + 6 + 3 m	21 m

- C1 17.5 cm per point.
- C2 10 times: 14 m is 1,400 cm, and 1,400 ÷ 140 = 10. It works for every column, and it is the rule the Sagrada Família itself gives: the shaft is ten times the inner diameter.
- C3 Because they carry more weight.

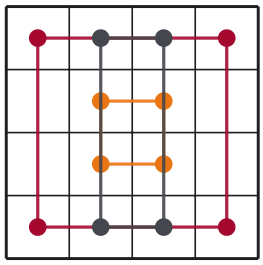
FURTHER READING

- Bonet i Armengol, J. (2000). *L'últim Gaudí = The Essential Gaudí*. Barcelona: Pòrtic. In Catalan and English.
- Burry, M. (ed.) (2007). *Gaudí Unseen: Completing the Sagrada Família*. Berlin: Jovis.

The full bibliography, with links, is at crm.cat/les-matematiques-de-gaudi.

ACTIVITY 6 · CHAPTER 08 · THE (ALMOST) MAGIC SQUARE

Your own (almost) magic square



On the Passion façade of the Sagrada Família there are sixteen numbers that add up to 33 along rows, columns and diagonals. The sculptor Josep Maria Subirachs put them there; they are not Gaudí's. Here you will see why his square can't be fully magic, and you will make one with your own date of birth.

Name _____ Class _____ Date _____

TIME	50 minutes
GROUPS	On your own or in pairs
YOU NEED	Pencil, colours and, if you need one, a calculator.

A Subirachs's square

A1 Add up each row, each column and both diagonals. Write the totals in the grey boxes, and the diagonals in the dashed ones.

A2 There are many more ways to pick four cells that add up to 33: the four corners, the four in the middle... Mark them by joining the cells with a coloured line, like the drawing at the top. There are 88 in all. How many can you find? .....

1	14	14	4	
11	7	6	9	
8	10	10	5	
13	2	3	15	

A3 Which numbers appear twice? _____ Which numbers from 1 to 16 are missing? _____

A4 A "real" 4 × 4 magic square uses the numbers 1 to 16, each exactly once. What do all sixteen add up to? (Hint: pair 1 with 16, 2 with 15... How many pairs are there, and what does each pair add up to?)

_____

If that total is shared out over four rows with equal sums, what must each row add up to? _____ Why couldn't Subirachs get to 33 without repeating numbers?

_____

_____

B Your birthday square

Write your date of birth as four two-digit numbers: A, the day; B, the month; C, the first two digits of the year; and D, the last two. For example, 15 March 2013 gives A = 15, B = 3, C = 20 and D = 13. Then fill in the empty square following the template.

The template

A	B	C	D
D + 1	C − 1	B − 3	A + 3
B − 2	A + 2	D + 2	C − 2
C + 1	D − 1	A + 1	B − 1

Example: 15-03-2013

15	3	20	13
14	19	0	18
1	17	15	18
21	12	16	2

Your square

B1 Check that the rows, columns and diagonals all add up to the same number. What is it? _____ Compare it with A + B + C + D. _____

B2 Do the four corners add up to the same number? And each of the four 2 × 2 squares in the corners?

B3 Why does it work? Look at the second row: D + 1, C − 1, B − 3, A + 3. What happens to the +1 and −1, and to the −3 and +3, when you add it all up? Do the same with the first column.

B4 You might get a zero or a negative number (if you were born in January, B − 3 = −2). That's fine: the square still works. Like Subirachs's, yours isn't a real magic square either. Why not?

GOING FURTHER

Make the square for Gaudí's birthday: he was born on 25 June 1852. What do the rows add up to? Are any numbers repeated? And another challenge: in the square Albrecht Dürer engraved in 1514, the bottom row hides the year of the engraving (15 14). Where is your date in your square?

Your own (almost) magic square

SOLUTIONS AND NOTES

AIMS

Practise calculation, use Gauss's pairing trick, understand why terms cancel in an expression and reason about why something is impossible.

MATHS TOPICS

Number · Algebra (expressions and equivalence).

TIPS

- In part A, groups can hunt for combinations and pool them on the board.
- In part B, anyone who would rather not use their own date can pick a relative's or a made-up one.

WHERE SUBIRACHS'S SQUARE COMES FROM

From Dürer's, which adds up to 34. Subirachs rotated it half a turn and took 1 away from four cells, one in every row, column and diagonal, so each line loses exactly 1. 33 is the age tradition gives Christ at his death.

THE BIRTHDAY SQUARE

The template is often called "Ramanujan's magic square", because it is usually shown with the birth date of the Indian mathematician Srinivasa Ramanujan, 22-12-1887.

SOLUTIONS

- A1 They all add up to 33.
- A2 The corners (1 + 4 + 13 + 15), the centre (7 + 6 + 10 + 10), the four 2 × 2 corner squares (1 + 14 + 11 + 7, 14 + 4 + 6 + 9, 8 + 10 + 13 + 2, 10 + 5 + 3 + 15), the middle cells of the top and bottom rows (14 + 14 + 2 + 3), the outer cells of the two middle rows (11 + 9 + 8 + 5)… Counting cells rather than values, there are 88 groups of four. Counting every group of three to seven cells (the 88 included) gives 310, the figure the Sagrada Família quotes.
- A3 10 and 14 appear twice; 12 and 16 are missing.
- A4 8 pairs of 17: 136. 136 ÷ 4 = 34. For 33 the total would have to be 132, and with the numbers 1 to 16 used once each it is always 136.
- B1 Always A + B + C + D. In the example, 51.
- B2 Yes: the corners and the four 2 × 2 squares, and the four central cells too.
- B3 Every row, column and diagonal has A, B, C and D once each, plus numbers that add up to zero (second row: +1 − 1 − 3 + 3; first column: +1 − 2 + 1).
- B4 Numbers can repeat, and they are not 1 to 16.
- Gaudí 25 + 6 + 18 + 52 = 101, and no number repeats.

To follow up: chapter 08 at crm.cat/les-matematiques-de-gaudi, with Subirachs's and Dürer's squares to play with; the Sagrada Família blog article "The magic square on the Passion façade"; and the National Gallery of Art (Washington) page for "Melencolia I".

FURTHER READING

- Weisstein, E. W. "Dürer's Magic Square" and "Magic Square". *MathWorld*, Wolfram Research.

The full bibliography, with links, is at crm.cat/les-matematiques-de-gaudi.