Contribution for "Homotopy structures in Barcelona"

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Title: Properadic aspects of Poincaré duality: Koszulity and formality.

Abstract: In noncommutative geometry, one studies algebraic avatars of geometric and topological structures. In the case of Poincaré duality, the classical perspective is to use Frobenius algebra structures, or their homotopy analogues, on algebras of cochains. In this talk I will present a Koszul-dual version of this story, which uses instead algebras of chains on based loop spaces, and a certain properadic algebra structure that we call Y_∞-algebra. This structure sits between pre-Calabi-Yau algebras and homotopy double Poisson algebras, and is encoded by a Koszul-contractible properad, in other words, its deformation theory is controlled by genus-zero information. I will present the general idea behind the proof of Koszulity of this properad, which uses the geometry of quadratic differentials, and discuss some formality results for Y_∞-algebras. This talk has material from preprints arXiv:2503.04297, joint with Coline Emprin, and arXiv:2511.02829.