LOCALIZATION OF ∞-OPERADS

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Algebraic topology may be viewed as the study of the interplay between geometric and algebraic structures. Since the foundational works of May and Boardman–Vogt, operads have emerged as a unifying and combinatorially accessible framework for encoding such interactions. In the context of homotopy theory, where spaces are replaced by their cellular approximations and categories by ∞-categories, the theory of operads admits a higher-categorical refinement in the form of ∞-operads. Their importance stems from three central features: they describe higher algebraic structures of objects of ∞-categories, they interact closely with symmetric monoidal ∞-categories, and the different models provide flexible and often complementary approaches for working homotopy-coherently.

A fundamental way in which ∞-operads naturally arise in homotopical contexts is through the process of localization: given an operad and a chosen class of morphisms, one may form the ∞-operad obtained by freely inverting these arrows. This mechanism plays a crucial role in examples such as factorization homology, where one studies E_n-algebras on the open subsets of a manifold. Localization is well understood at the level of ∞-categories: Joyal's delocalization theorem asserts that every ∞-category is weakly equivalent to the localization of a discrete category, while the classical work of Barwick and Kan shows that this process establishes an equivalence at the level of homotopy theories, between that of ∞-categories and that of categories equipped with weak equivalences.

In this talk, I will present a generalization of Joyal's delocalization theorem to the operadic setting, showing that every ∞-operad is weakly equivalent to the localization of a discrete operad. The proof is combinatorial in nature, relying on the analysis of trees in the dendroidal category. Building on this result, we explore its applications to the homotopy theory of ∞-operads. For instance, in joint work with Arakawa and Carmona, we show that the homotopy theory of discrete operads with weak equivalences is equivalent, via localization, to that of ∞-operads, generalizing the result of Barwick and Kan. Time permitting, I will also discuss ongoing work on the construction of models for operadic localization.