title: An inductive approach to the Diaz-Park sharpness conjecture

abstract: Fusion systems are categories that generalize the p-local structure of finite groups and have applications in various areas such as modular representation theory and homotopy theory. A long standing question concerning the homotopy theory of fusion systems was formulated in 2014 by Diaz and Park and is known as the "*sharpness conjecture for fusion systems*". This conjecture states that, given a p-group S, a saturated fusion system $\mathcal{F}$ over S and a Mackey functor $M = (M_*, M^*)$ over $\mathcal{F}$ with coefficients in $\mathbb{F}_p$ (as for example the cohomology Mackey functor with $M^* = H^n(-, \mathbb{F}_p)$) then $\lim^i_{\mathcal{O}(\mathcal{F}^c)} M^* = 0$ for every $i \geq 1$. This vanishing of higher limits is known to hold whenever there exists a finite group G such that $S \in \mathrm{Syl}_p(G)$ and that the morphisms in $\mathcal{F}$ coincide with the morphisms given by conjugation with an element of G (i.e. $\mathcal{F}$ is realizable and $\mathcal{F} = \mathcal{F}_S(G)$). However, outside of specific cases, it is not known if the conjecture holds when such a group G doesn't exist (i.e. $\mathcal{F}$ is exotic). It follows that a counterexample to the sharpness conjecture would provide a tool (other than the classification of finite simple groups) for distinguishing exotic and realizable fusion systems. On the other hand, if proven true, the sharpness conjecture could provide a simple proof of the stable elements theorem and, hopefully, deepen our understanding of exotic fusion systems. Studying this conjecture can therefore be of interest regardless of the outcome. This interest has been reflected in recent years by a variety of publications studying the sharpness conjecture. Interestingly enough the results presented in most of these publications depend, at least indirectly, on a previous result of Jackowski and McClure which provides vanishing of higher limits for what they call proto-Mackey functors.

During this talk we will briefly introduce fusion systems and the sharpness conjecture. We will then present our approach to the conjecture which, instead of relying on the result of Jackowski and McClure, makes use of a novel inductive approach. We will then conclude by listing different families of fusion systems for which this novel approach can be applied to answer the sharpness conjecture.