Finite partial groups are genuinely finite

Rémi Molinier

Partial groups are, roughly speaking, groups in which the product of a given word of elements may not always be defined. They were introduced by Chermak to study the p-local structure of finite groups and come equipped with a "domain," which consists in the set of words for which the product is defined. At first glance, given a fixed set X, it may seem possible to define infinitely many partial group structures on X by varying the domains, even while ensuring that all these structures have coherent products. For example, if G is a finite group, one might expect that there could be infinitely many different partial group structures "contained" in G by selectively removing sets of words from the domain.

However, in a joint work with Philip Hakney, we show that finite partial groups are truly finite objects: they can be defined using only a finite set of data and, in particular, contain only finitely many "partial subgroups." This follows from the fact that finite partial groups have finite dimension as symmetric sets. During the talk, we will explore both the algebraic and topological approaches to partial groups and no prior knowledge of the topic will be assumed.