Quillen-Barr-Beck cohomology for divided power algebras over an operad [3]*

Sacha Ikonicoff

Université de Strasbourg

In his thesis [2], Beck defined a general notion of modules and derivations in a sufficiently nice category. Using similar methods, Quillen introduced a cohomology theory for commutative rings, known as the André-Quillen cohomology, by studying something called the cotangent complex for these rings [8, 1]. Combining both methods allows us to build a cohomology theory for a large family of categories, known as the Quillen–Barr–Beck cohomology.

An operad is a device which encodes types of algebras [7]. We can also use operads to define categories of divided power algebras, which have additional monomial operations [4]. These divided power structures appear notably in the simplicial setting. The aim of this talk is to show how Quillen cohomology generalises to several categories of algebras using the notion of operad. We will introduce modules and derivations, but also a representing object for modules – known as the universal enveloping algebra – and for derivations – known as the module of Kähler differentials – which will allow us to build an analogue of the cotangent complex. We will see how these notions allow us to recover known cohomology theories on many categories of algebras [6, 5], while they provide somewhat exotic new notions when applied to divided power algebras [3].

References

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