TOWARDS A THEORY OF CHARACTERISTIC CLASSES FOR STRATIFIED SPACES

BRUNO GALVEZ ARANEDA

Characteristic classes of vector bundles are a powerful organizing tool in topology and geometry: they control obstructions to the existence of additional structures, detect subtle features of maps, and enter systematically in index theory, Riemann–Roch type theorems, and the study of manifold invariants. In the classical smooth setting these classes are attached functorially to vector bundles (most notably the tangent bundle) and admit concrete geometric realizations, together with a rigid package of formal properties such as naturality, multiplicativity, and compatibility with products and fundamental classes. However, many spaces of genuine geometric interest are singular, and one would like to retain a theory of characteristic classes that still reflects the geometry of such spaces. This has led, over several decades, to a variety of constructions of characteristic classes for singular spaces (MacPherson's Chern classes, L–classes, etc.), typically formulated in terms of resolutions, embeddings, or extra geometric data. Conceptually, it remains unsatisfactory that there is no intrinsic "tangent like" object on a stratified space playing the universal role of the tangent bundle and directly controlling characteristic classes in a homotopy-theoretic framework adapted to singularities.

In this talk, I will describe work in progress that addresses this issue by combining the homotopical study of stratified spaces with intersection homology. First, I will introduce a notion of a stratified vector bundle on a stratified space. Such a bundle is allowed to have different ranks on different strata, subject to compatibility conditions along links between strata. I will show how these objects are classified by maps into a stratified classifying space, constructed using the diagrammatic approach to the homotopy theory of stratified spaces, and I will compute the intersection cohomology of this object. This framework allows us to construct examples such as the tangent bundle and the Thom space of a stratified space. Second, I will place intersection cohomology into a bivariant framework, in the sense of Fulton–MacPherson, by working with a spectral enhancement of the sheaf-theoretic aproach of intersection cohomology, in order to address several structural and functorial issues. If time permits, I will show how the combination of these two ingredients leads us to a theory of characteristic classes for stratified spaces.