

Matroids arising from algebraic shifting

The operator of algebraic shifting, introduced by Kalai, associates with any simplicial complex a shifted one, i.e. $i < j \in F \setminus K$ implies $(F \setminus \{j\}) \cup i \in K$, which is combinatorially simpler. Algebraic shifting preserves topological invariants of the initial complex, such as f-vector and Betti-vector, as well as encodes some of its combinatorial properties. There are different variations of algebraic shifting, we will mainly focus on exterior shifting and use it in the setting of uniform hypergraphs. We call a uniform shifted hypergraph matroidal if its preimage under exterior shifting is the set of bases of some matroid. There are several examples of known matroids arising in this way: simplicial matroid, hyperconnectivity matroid and area-rigidity matroid. We characterise all matroidal simple graphs and 3-uniform hypergraphs. Namely, such a hypergraph is matroidal if and only if its hyperedges form an initial segment w.r.t. the lexicographic order. In the k-uniform case we provide a similar characterisation for shifted hypergraphs satisfying some additional conditions.

Joint work with Eran Nevo.