

Inductive construction of deformed permutahedra

A new bound on the number of rays of the submodular cone

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The submodular cone is the cone of deformed permutahedra (*a.k.a.* submodular functions, generalized permutahedra, polymatroids). We present an inductive construction of the submodular cone, using an operation called the *GP-sum*: from two deformed permutahedra in dimension n , we create (bijectively) one deformed permutahedra in dimension $n + 1$.

Empowered by this construction, we create new rays of the submodular cone, *i.e.* new Minkowski indecomposable deformed permutahedra. We improve the bounds on the number of these rays: the n^{th} submodular cone has at least 2^{2^n} rays. We also obtain bounds (upper and lower) on f -vector of the submodular cone, on the total number of its faces, and on the number of faces which are simplicial cones.