Hamiltonicity of Simplicial and Supersolvable Arrangements

Veronika Körber[2], Tobias Schnieders[2], Jan Stricker[1], Jasmin Walizadeh[2]

[1]) Person contributing talk
stricker@math.uni-frankfurt.de
[2]) Coauthors

There are a lot of algorithms, which construct all permutations successively. The sequences of permutations produced by some of these algorithms can be interpreted as Hamilton cycles of the tope graph of the braid arrangement. Conway, Sloane, and Wilks expanded this perspective to all reflection arrangements and constructed Hamilton cycles for their tope graph. In general, the tope graph of a hyperplane arrangement does not admit a Hamilton cycle.
In this talk I will report on new results on the existence of Hamilton cycles in the tope graphs of all known 3-dimensional *simplicial*, all restrictions of reflection arrangements and *supersolvable* arrangements, extending known results to larger families beyond classical reflection arrangements.
We begin by investigating the Hamiltonicity of all known 3-dimensional simplicial arrangements and present constructions of Hamilton cycles for all their infinity families as well as all known exceptional arrangements.
The *restrictions of reflection arrangements*, which are not necessarily reflection arrangements themselves, inherit enough combinatorial structure to facilitate constructive proofs.
In the second part of the talk, we turn to *supersolvable arrangements*, which are not necessarily simplicial arrangements. These hyperplane arrangements play a significant role in the study of free arrangements. Leveraging a deep structural decomposition result from Björner, Edelman, and Ziegler, we develop an inductive framework that guarantees Hamiltonicity for all supersolvable arrangements. Notably, this framework extends naturally to *supersolvable oriented matroids*.
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