GENERALIZED BREAK DIVISORS AND TRIANGULATIONS OF LAWRENCE POLYTOPES

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Abstract. Let G be a connected graph of genus g. The Picard group of degree g, $\mathrm{Pic}^g(G)$, is the set of equivalence classes of divisors on G of degree g, where two divisors are equivalent if one can be reached from the other through a sequence of chip-firing moves. One canonical choice of representatives of $\mathrm{Pic}^g(G)$ is the *integral break divisors*, which are induced by the spanning trees T of G in the following way: for a spanning tree T, orient the edges of G not in T, and place a chip at the head of each oriented edge. Every degree g divisor on G is equivalent to a unique divisor of this form.

However, the break divisors are only one choice for representatives of $\mathrm{Pic}^g(G)$. In this talk, we will construct sets of representatives of the equivalence classes in $\mathrm{Pic}^g(G)$, called *generalized break divisors*, by constructing a function I_G on the spanning trees of G from a triangulation of the Lawrence polytope of the cographic matroid $\mathcal{M}^*(G)$. If $\mathcal{D}(T,\mathcal{O})$ is a break divisor induced by a spanning tree T, then $I_G(T)+\mathcal{D}(T,\mathcal{O})$ is a generalized break divisor. The set of generalized break divisors with respect to a function I_G is denoted by $\mathcal{BD}_{I_G}(G)$. For each I_G that we construct, every degree g divisor on G is equivalent to a unique generalized break divisor of the form $\mathcal{D}(T,\mathcal{O})+I_G(T)$.

Additionally, all I_G such that $\mathcal{BD}_{I_G}(G)$ is a set of representatives of the equivalence classes in $\mathrm{Pic}^g(G)$ correspond to stability conditions on the nodal curve dual to the graph G. In this talk, we will show that I_G that are constructed from regular triangulations of Lawrence polytope of the cographic matroid $\mathcal{M}^*(G)$ correspond to classical stability conditions, which are induced by generic real-valued divisors on G.