

Computing Chow classes of matroids by enumerating Young Tableaux

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We study the Chow classes of matroids in the Chow ring of the Grassmannian, a generalization of the torus orbit closures of points under the action of the maximal torus. An intriguing conjecture states that their expansion in the Schubert cycle basis only has positive coefficients. While this is immediate in the realizable case, it has been proved in only a few special instances and remains widely open in general, largely due to the complexity of the existing formulas.

We develop an identity describing how Chow classes behave under series and parallel extensions of matroids. This relation enables a recursive computation for a fundamental family of matroids called snake matroids, which are particular lattice path matroids associated with ribbon-shaped diagrams. Our main result states that the Poincaré dual of the Chow class of a snake matroid corresponds to a ribbon Schur function. Combined with results on the valuativity of this class under matroid polytope subdivisions, this leads to an algorithmic formula for computing Chow classes of arbitrary matroids.

This correspondence transforms the geometric problem of determining Schubert coefficients into a purely combinatorial one: for every lattice path matroid, each coefficient counts certain standard Young tableaux with a prescribed descent set. This insight not only sheds new light on the structure of these classes, but also yields simple proofs of classical results such as Klyachko’s formula for uniform matroids, the Gessel–Viennot enumeration of permutations with fixed descent sets, and volume formulas for lattice path matroids. Beyond these results, we also establish new partial nonnegativity results for the class of paving matroids.

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