Shellings of Bergman Fans

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1 Abstract

Given a matroid M and a building set B on its lattice of flats, we prove that the associated nested set complex N is shellable. This generalizes a classical result of Björner that the order complex of the lattice of flats of a matroid is shellable (the case when B is the maximum builiding set), and strengthens a result of Feichtner-Müller that N is Cohen-Macaulay for arbitrary B. Our approach is geometric in nature utilizing the Bergman fan $\Sigma_{M,B}$, and is inspired by Bruggesser and Mani's line shellings of polytopes. We prove that, given a normal complex P for $\Sigma_{M,B}$, as introduced by Ross and the third author, and a particularly well-behaved vector ω, the order of the vertices of P induced by ω is a shelling order for N. This is joint work with Spencer Backman, Anastasia Nathanson, Ethan Partida, and Noah Prime.