Title: The f-vector conjecture via h* vectors of section rings

Abstract:

This talk is based on joint work with Chris Eur and Matt Larson, arXiv:2510.05207.

Speyer's 2005 f-vector conjecture asserted the nonnegativity of the coefficients of a matroid invariant he defined, notably the leading coefficient omega(M). When M is represented by a hyperplane arrangement, Larson identified omega(M) as the Euler characteristic of a certain anti-nef line bundle L^-1 on the wonderful compactification of the arrangement, up to a predictable sign. Eur and Larson used this to show that omega(M) is the leading coefficient in the "h* polynomial" of L, i.e. the numerator of the Hilbert series of the total coordinate ring of the image of the map to projective space defined by L. If the image is sufficiently nice (arithmetically Cohen-Macaulay), then the h* polynomial necessarily has nonnegative coefficients.

The work I'll be talking about completes the argument, giving a second proof of the f-vector conjecture. We show that wonderful varieties degenerate inside the permutahedral toric variety to a Cohen--Macaulay union of torus orbits controlled by a second matroid. This union of torus orbits can be defined when M is not representable, and the needed Euler characteristic can be computed on the degeneration.