Unimodular triangulation for s-lecture hall simplices

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Abstract

Given a sequence of positive integers $\boldsymbol{s} = (s_1, \ldots, s_n)$, the s-lecture hall simplex is defined as:

$$\mathcal{P}_n^{\boldsymbol{s}} \coloneqq \left\{ \boldsymbol{x} \in \mathbb{R}^n \ : \ 0 \leq \frac{x_1}{s_1} \leq \cdots \leq \frac{x_n}{s_n} \leq 1 \right\}.$$

Introduced by Savage and Schuster [SS12], s-lecture hall simplices have been widely studied in discrete geometry and combinatorics.

A central question in discrete geometry is whether $\mathcal{P}_n^{\boldsymbol{s}}$ admits a unimodular triangulation, *i.e.,* a triangulation into lattice simplices of unit volume. It is conjectured that $\mathcal{P}_n^{\boldsymbol{s}}$ admits a unimodular triangulation for every $\boldsymbol{s}$ [HOT16, Conj. 5.2]. While some progress has been made toward resolving this conjecture, significant gaps remain. In particular, [HOT16, Thm. 3.3] proves that $\mathcal{P}_n^{\boldsymbol{s}}$ admits a unimodular triangulation if $s_{i+1} = k_i s_i$ for some $k_i \geq 1$, and [BS20, Cor. 3.5] shows the existence of regular-unimodular triangulations when $0 \leq s_{i+1} - s_i \leq 1$. In this talk, I present a significant extension of these results by establishing a more general (and recursive) criterion for the existence of unimodular triangulations of $\mathcal{P}_n^{\boldsymbol{s}}$: Let $\boldsymbol{s} = (s_1, \ldots, s_{n-1})$ and $s_n = k\, s_{n-1} + \varepsilon$ with $k \geq 1$ and $\varepsilon \in \{0, 1\}$. If $\mathcal{P}_{n-1}^{\boldsymbol{s}}$ admits a unimodular triangulation, then $\mathcal{P}_n^{(\boldsymbol{s}, s_n)}$ also admits one.

This is joint work with Martina Juhnke and Germain Poullot.

References

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