

Title: Higher-Categorical Associahedra

Abstract: In 2017, Bottman introduced a family of posets called 2-associahedra as a tool for investigating functoriality properties of Fukaya categories, and he conjectured that they could be realized as face posets of convex polytopes. We introduce a family of posets called categorical n -associahedra, which naturally extend Bottman's 2-associahedra and the classical associahedra. Categorical n -associahedra give a combinatorial model for the poset of strata of a compactified real moduli space of a tree arrangement of affine coordinate subspaces. We construct a family of complete polyhedral fans, called velocity fans, whose coordinates encode the relative velocities of pairs of colliding coordinate subspaces, and whose face posets are the categorical n -associahedra. In particular, this gives the first fan realization of 2-associahedra. In the case of the classical associahedron, the velocity fan specializes to the normal fan of Loday's realization of the associahedron. This is joint work with Nathaniel Bottman and Daria Poliakova.