Rigidity theory for symmetric frameworks - *Alison La Porta*

Rigidity theory is an area of mathematics which uses tools from graph theory, combinatorics and matroid theory (amongst others) to determine whether a given structure, the 'framework', is 'rigid' or 'flexible'. There are various mathematical notions of *framework*, the most standard of which sees it as a pair (G, p), where G is a graph and $p : V(G) \to \mathbb{R}^d$ is a map.

There are also many notions of rigidity. Most intuitively, a framework is *rigid* if it cannot be continuously deformed without changing its edge-lengths, and it is *flexible* otherwise. A slightly stronger notion of rigidity, *infinitesimal rigidity*, considers velocity vectors assigned to $V(G)$, rather than continuous displacements of the vertices. In particular, this linearises the problem.

A framework is said to be *generic* if $\{p(v)\}_{v \in V(G)}$ is algebraically independent over $\mathbb{Q}$. Assuming genericity, rigidity and infinitesimal rigidity coincide, and they can both be treated as a property of the underlying graph G. Notably, the *Geiringer-Laman theorem* combinatorially characterises generically rigid graphs in the Euclidean plane. The analogous characterisation fails for higher dimensions.

If (G, p) presents a non-trivial symmetry, unexpected behaviour can occur. Symmetric frameworks have received significant interest for over two decades, as symmetry occurs naturally in many application areas. The research splits into the study of *forced* and *incidental* rigidity. In both cases, the framework starts in a symmetric position. However, in the former, the symmetry of the framework must be maintained throughout its motions, whereas in the latter the framework can move in unrestricted ways.

Assuming symmetry-adapted genericity, forced and incidental rigidity can again be treated as properties of G. Under this assumption, I have combinatorially characterised the incidental rigidity of plane frameworks which are symmetric with respect to a cyclic group of order 2,4,6, or of odd order less than 1000. This was part of my PhD research, and it extends previous results in which the symmetry group action was assumed to be free on $V(G)$ (e.g. no vertices on the symmetry line of a reflection-symmetric frameworks) - a generalisation required the introduction of new geometric and combinatorial tools. This is collaborative work with Bernd Schulze.

Suppose now that (G, p) is a realisation of G in the upper-half plane, so that $p : V(G) \to \mathbb{H}$. Then, the geometric difficulty of the problem changes significantly, though some the combinatorial techniques can be adapted to this setting. In a recent research project, we combinatorially characterised the forced rigidity Γ-symmetric frameworks in $\mathbb{H}$, where Γ is a Fuchsian group. (Here, we assumed both symmetry-adapted genericity and a free symmetry-group action on $V(G)$.) This work is a collaboration with Sean Dewar, Rebecca Monks, Anthony Nixon, Klara Stokes and Joannes Vermant.