BIFURCATION ANALYSIS OF A CROSS-DIFFUSION SYSTEM FOR GANG TERRITORIALITY

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We investigate a cross-diffusion system modeling gang territorial behavior, based on the continuum model introduced in [AB18]. The model describes two rival gangs, A and B, which compete over territory using graffiti markings as indirect communication. The presence of each gang is represented by a density function, and graffiti fields evolve based on how agents mark their territory.

Assuming that graffiti dynamics equilibrate much faster than the movement of agents, the original four-equation systemis reduced to a two-species cross-diffusion model involving only the gang densities ρ_A and ρ_B. To introduce asymmetry and enable bifurcation analysis, we add a linear decay term to one equation.

Our main focus is to analyze how the bifurcation parameter β, which encodes the strength of inter-gang avoidance, influences the emergence of non-homogeneous steady states. Using the Crandall–Rabinowitz theorem, we show that spatially homogeneous steady states lose stability when β is outside the certain parameter regime.

In particular, we derive an explicit formula for the bifurcation points.

These analytical results are complemented by numerical continuation. The findings demonstrate that, for specific parameter values, the system admits non-homogeneous solutions — confirming results previously obtained for the corresponding discrete model.

References

[AB18] Abdulaziz Alsenafi and Alethea B. T. Barbaro. "A convection-diffusion model for gang territoriality". In: *Physica A. Statistical Mechanics and its Application* (2018). DOI: https://doi.org/10.1016/j.physa.2018.07.004.