Indirect Influence on Network Diffusion

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Abstract

Models of network diffusion typically rely on the Laplacian matrix, capturing interactions via direct connections. Beyond direct interactions, information in many systems can also flow via indirect pathways, where influence typically diminishes over distance. In this work, we analyze diffusion dynamics incorporating such indirect connections using the d-path Laplacian framework. We introduce a parameter, the indirect influence, based on the change in the second smallest eigenvalue of the generalized path Laplacian, to quantify the impact of these pathways on diffusion timescales relative to direct-only models. Using perturbation theory and mean-field approximations, we derive analytical expressions for the indirect influence in terms of structural properties of random networks. Theoretical predictions align well with numerical simulations, providing a phase diagram for when indirect influence becomes significant. We also identify a structural phase transition governed by the emergence of d-paths and derive the critical connection probability above which they dramatically alter diffusion. This study provides a quantitative understanding of how indirect pathways shape network dynamics and reveals their collective structural onset.