Anticipating explosive synchronization in networks of Kuramoto oscillators

Juan Gancio,[1] Giulio Tirabassi,[2] Cristina Masoller,[1]

1. Universitat Polintècnica de Catalunya, Departament de Fisica, Rambla Sant Nebridi 22, Terrassa 08222, Barcelona, Spain

2. Universitat de Girona, Departament de Informàtica, Matemàtica Aplicada i Estadística, Carrer de la Universitat de Girona 6, Girona 17003, Spain.

Many complex systems experience sudden changes in their dynamics as they transition between different states when a control parameter is varied. The threshold value of the parameter for which an abrupt transition occurs is know as "tipping point". For real–world systems, such as animal populations, vegetation fields, and the climate, there is a particular interest in being able to detect if a tipping point is approaching, since once crossed the systems may not recover the previous stable state [1]. An example of a sudden change is explosive synchronization (ES), an abrupt transition between a disorder state and a synchronous one, characterized by being of first–order [2].

In this work, we propose several quantifiers based on the permutation entropy (PE) that present a particular behavior when the network approaches a synchronous state. PE is a time series analysis technique that uses symbols (ordinal patterns) defined by the ordering of the data points, disregarding the actual values [3], and has recently been proposed as a new indicator for critical transitions [4,5]. We test this new method in different networks of Kuramoto phase oscillators linked in such a way that they present ES. We observe substantial and characteristic changes in these quantities as the networks approach synchronization, even when the synchronization is explosive. Therefore, PE-based quantities can serve as early warning signals (EWS) of this class of transitions. Our results complement previous findings regarding PE methods for EWS of critical transitions, expanding their applications to systems characterized by complex temporal behaviors.

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