Effect of kinetics on non-equilibrium phases of a driven Potts model

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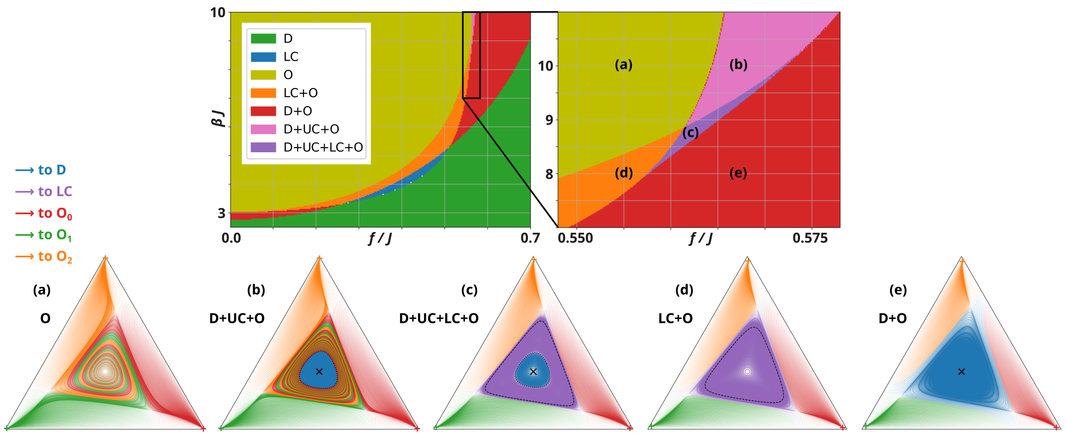

The Potts model is a generalization of the Ising model where spins can take more than two states. We study a driven three-state nonequilibrium Potts model with homogeneous all-to-all coupling. This is the minimal model, which at the mean-field level exhibits complex behavior, such as synchronization that leads to the emergence of persistent macroscopic oscillations (limit cycles). (*1*)

We first consider the mean-field limit, where the model dynamics is described via deterministic equations of motion for macroscopic variables of the system. The main result is that the choice of the transition rate function can drastically reshape the phase diagram of the model, generating much richer behavior than that originally reported in Ref. (*1*). We identified seven distinct dynamical phases separated by six bifurcation types: super- and subcritical Hopf, heteroclinic, infinite period, saddle-node, and saddle-node bifurcation of limit cycles. Specifically, we observed a disordered phase (D), an ordered phase (O) with three symmetric point attractors, a lone limit cycle (LC), and multi-stable phases with more than one attractor type, sometimes separated by an unstable cycle (UC): D+O, D+UC+O, LC+O, D+UC+LC+O.

Beyond the mean-field analysis, we further characterize the effect of rare fluctuations on the model behavior. First, using the method of Gaspard (*2*), we determine the coherence lifetime of the oscillations and compare it to a thermodynamic bound given by the entropy production per cycle. (*3*, *4*) In this way, we show that the trade-off between coherence lifetime and entropy production can be tailored by adjusting the parameters of the transition rate model.

Secondly, using the instanton approach (*5*, *6*), we characterize the rare stochastic transitions among the coexisting mean-field attractors, which tend to relax the system to a single "most likely" attractor that determines the macroscopic behavior of the model. In particular, we show that in the region of multistability, the ordered state is usually more likely than the D or LC state.

References

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