A Bayesian approach to mesoscale patterns: disentangling nestedness, modularity and in-block nestedness

HR de los Ríos,[1,*] A Solé-Ribalta,[1] M Sales-Pardo,[2] R Guimerà,[2] and J Borge-Holthoefer[1]

[1] *Internet Interdisciplinary Institute (IN3), Universitat Oberta de Catalunya, Barcelona, Catalonia, Spain*
[2] *Department of Chemical Engineering, Universitat Rovira i Virgili, 43007 Tarragona, Catalonia, Spain*

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Complex networks often simultaneously exhibit modularity –densely connected communities– and nestedness, where interactions form hierarchical subsets. Although prevalent across ecological, economic, and social systems, these architectures have long been considered incompatible [1] because competitive dynamics that foster modularity appear to contradict the cooperative mechanisms underlying nestedness. Current methodological limitations exacerbate this tension: global nestedness measures disregard community structure, whereas community-detection algorithms assume intra-block homogeneity. Consequently, existing techniques systematically miss architectures that are both modular and hierarchically nested.

The concept of in-block nestedness (IBN) was recently introduced to reconcile this apparent incompatibility by quantifying nested organization confined within modules [2]. Detection of IBN, however, still relies on deterministic optimization heuristics [3] whose statistical meaning remains unclear, providing no assessment of uncertainty. Moreover, no generative model capable of producing—and rigorously inferring—IBN structures has been proposed thus far.

Here, we close this gap by formulating the Bayesian Nested Block Model (BNBM), the first probabilistic model jointly capturing modular and nested architectures. BNBM extends stochastic block models by embedding a latent hierarchical inclusion process inside each block. Each block α is characterized by a nested connectivity pattern parameterized by a continuous shape parameter ξ_α, while structural variability arises from two noise processes: intra-block perturbations p_α, which randomly relocate internal links, and global inter-block noise μ, redistributing links across blocks. The latent configuration of the model includes block memberships z, node orderings within blocks π, nestedness profiles ξ, both noise levels (p, μ), and the number of blocks B. Given parameters $\theta = \{z, \pi, \xi, p, \mu, B\}$, the model specifies edge-probability matrices $P_{ij}(\theta)$, allowing calculation of the network likelihood:

$$\mathcal{L}(A \mid \theta) = \prod_{i,j} P_{ij}(\theta)^{A_{ij}} \left[1 - P_{ij}(\theta)\right]^{1-A_{ij}} .$$

Inference is fully Bayesian and performed through hybrid Markov-chain Monte Carlo (MCMC), sampling the posterior distribution $P(\theta \mid A)$ to quantify uncertainty in block structure, nestedness, and model parameters. The MCMC algorithm iteratively updates parameters via symmetric Gaussian perturbations, random node permutations, and structural block moves including merging, splitting, and node reassignment using Gibbs sampling.

Model selection is based on the Bayesian Minimum Description Length (MDL). Figure 1 shows MDL differences between inferred and true structures across synthetic benchmarks, highlighting critical thresholds in noise parameters beyond which structural recovery becomes ambiguous. We record the minimal-MDL solution from posterior sampling as a representative configuration, while posterior predictive checks confirm that inferred models accurately reproduce structural patterns. Synthetic benchmarks demonstrate BNBM accurately recovers modular-nested structures even under significant noise.

When applied to empirical networks, our model also consistently uncovers clear modular-nested structures, demonstrating its robustness beyond synthetic cases. Crucially, the Bayesian MDL criterion effectively balances complexity and fit even in realistic scenarios, providing stable and interpretable results. Posterior predictive checks confirm that inferred structures capture essential empirical patterns.

Our unified Bayesian approach thus significantly advances understanding of multiscale organization in complex networks, offering a principled statistical framework for capturing coexistence of modularity and nestedness.

* hdelosrios@uoc.edu

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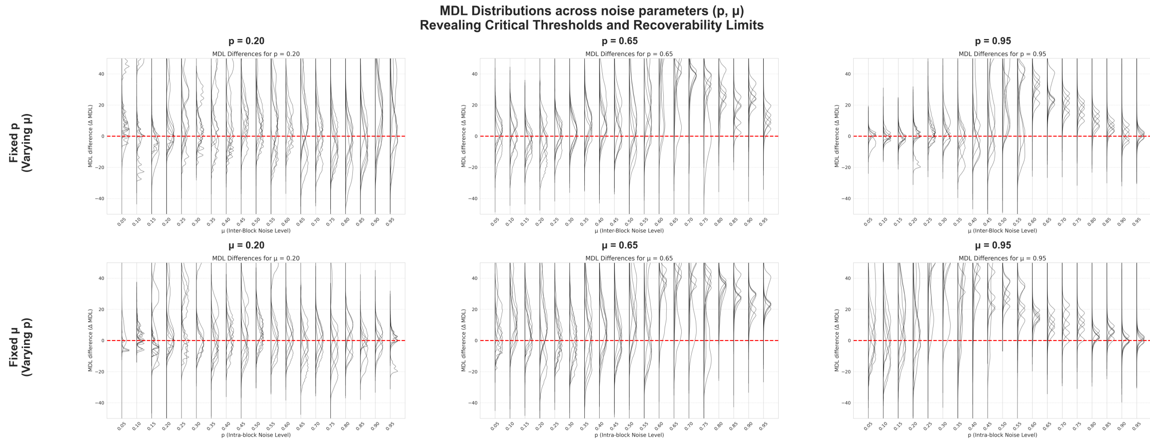

FIG. 1. MDL differences across synthetic benchmarks, illustrating recoverability limits under varying intra-block noise (p) and inter-block noise (μ). Red dashed lines indicate perfect recovery ($\Delta\mathrm{MDL} = 0$).