

Statistical Mechanics of Hyperbolic Random Graphs

The family of network models in hyperbolic space represents one of the most advanced frameworks for modeling the structure of real complex systems, whose connectivity is sparse, small-world, heterogeneous, highly clustered, and scale-invariant under network renormalization techniques. These geometric models exhibit other intriguing properties, like an anomalous, temperature-dependent phase transition between a geometric and a non-geometric phase. In the unweighted case, where links are either present or absent, the model can be derived within a statistical mechanics framework by maximizing the entropy of graph ensembles subject to constraints imposed by observations, with links effectively behaving as fermionic particles. Our approach provides the least biased prediction of network properties and establishes a principled framework for analyzing network structures, offering new perspectives and powerful analytical tools for both theoretical and empirical studies.