Exploring Pedestrian Permeability in Urban Sidewalk Networks

Robert Benassai-Dalmau,[1] Javier Borge-Holthoefer,[1] Albert Solé-Ribalta,[1]

1. Universitat Oberta de Catalunya, Rambla del Poblenou, 156, 08018 Barcelona, Spain.

The spatial structure of cities significantly influences pedestrian mobility, a crucial factor in designing sustainable urban environments. However, while pedestrian movement and walkability have been widely studied, quantitative insights into how urban layouts shape pedestrian flows remain limited. In this work, we study pedestrian diffusion through the lens of discrete vector fields to better understand the effects network structures may impose on walker dynamics.

To spatially characterize these dynamics, we leverage two well-established random walk models: the classical discrete-time random walk (DTRW) and the continuous-time random walk (CTRW) models. The CTRW model allows the incorporation of a temporal perspective into the random walk dynamic model by introducing a Poisson-distributed random variable, with exponentially distributed waiting time, to each network's node representing the rate of walkers transitioning to the next vertex [1]. In our context, to model the transition rates, the waiting times are adjusted based on the duration a walker spends traversing each link (street segment). Within the CTRW framework, this is modeled through a transition matrix R_{ij}, representing the rate between node i and j. With this framework, and given the net flow on edge (i, j) over the time interval dt, $d\omega_{ij} = [p_i(t)R_{ij} - p_j(t)R_{ji}]dt$, we can characterize the pedestrian vector field as the overall net flow of walkers up to a certain time, $t_{\max}$,

$$\omega_{ij} = \int_0^{t_{\max}} [p_i(t)R_{ij} - p_j(t)R_{ji}]\, dt. \tag{1}$$

We highlight that, in the stationary state, there is no net flux in any edge of the network, according to the detailed balance condition $p_i R_{ij} = p_j R_{ji}$, so the sum converges at the mixing time.

We then decompose the resulting field using the Helmholtz-Hodge Decomposition (HHD), which separates the vector field into gradient and solenoidal (cyclic) components, offering insights into pedestrian permeability over the transportation network.

By comparing the HHD components generated with those obtained simulating random pedestrians walking at a constant velocity, we demonstrate that CTRWs provide a suitable framework for analysis, as they allow the incorporation of edge lengths and pedestrian speeds at the model (inset of Fig. 1). CTRWs capture detailed influences of network geometry on movement patterns that traditional discrete-time random walk models cannot describe. Observe in the central panel of Fig. 1 how the highest node potentials are located on the periphery of this central-periphery lattice, while the lowest ones are in the central cluster. In contrast, results considering the DTRW display the opposite pattern. Theoretically, we highlight that the field is fully characterized by its gradient component, with cyclic components only emerging in simulations due to stochasticity. Overall, our results indicate that regions with shorter edge lengths and more intricate network structures enhance pedestrian permeability and diffusion, aligning with urban theories on walkability and accessibility, such as those proposed by Jane Jacobs. Showcasing a practical application, we analyze the sidewalk networks of Barcelona (right panel of Fig. 1), Paris, and Boston, detailing how variations in local geometry, street density, and connectivity shape pedestrian mobility and urban permeability. The CTRW framework offers an analytical tool to identify areas that either facilitate or constrain pedestrian movement. We hope our findings will aid in designing pedestrian-friendly cities by highlighting the critical role of structural layout in influencing movement dynamics and urban permeability. A complete version of the work can be found in [2].

[1] J. Petit, R. Lambiotte, and T. Carletti, "Classes of random walks on temporal networks with competing timescales," *Applied Network Science*, vol. 4, no. 1, p. 72, 2019.

[2] R. Benassai-Dalmau, J. Borge-Holthoefer, and A. Solé-Ribalta, "Exploring pedestrian permeability in urban sidewalk networks," *Chaos, Solitons & Fractals*, vol. 194, p. 116114, 2025.

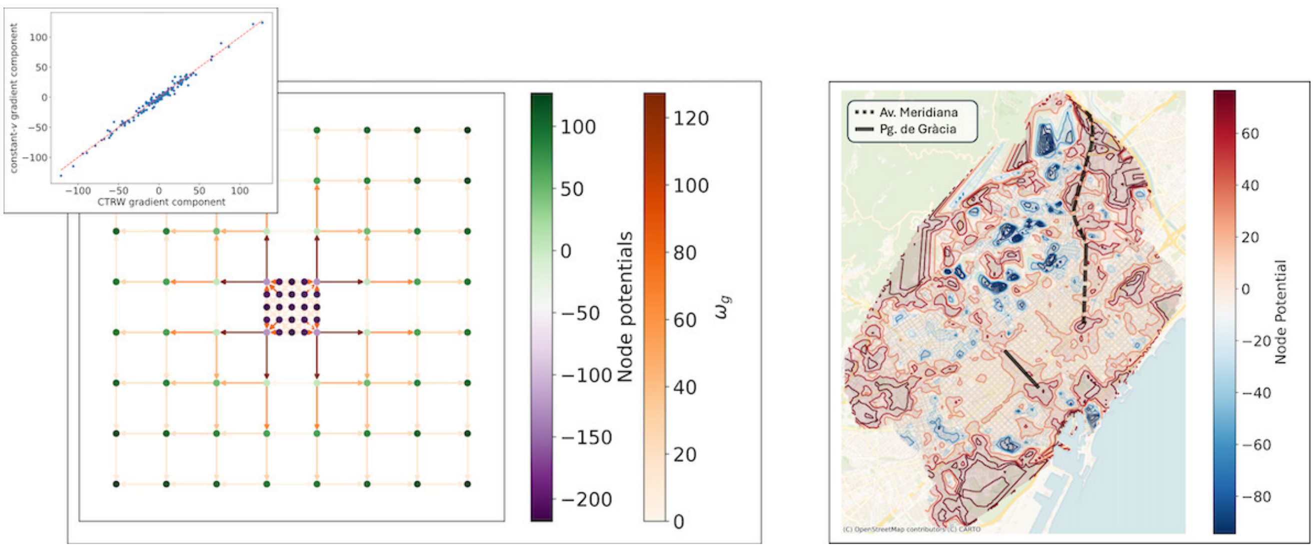

Figure 1: **(Inset)** Correlation between the CTRW and pedestrian simulations. **(Center)** Node potentials and gradients obtained with the CTRW in a central-periphery lattice. **(Right)** Node potentials obtained for the city of Barcelona.