Ergodicity Breaking and High-Dimensional Chaos in Random Recurrent Networks

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The random recurrent neural network model introduced by Sompolinsky, Crisanti, and Sommers (SCS) nearly four decades ago [1] has become a paradigmatic framework for studying complex dynamics in random recurrent networks. In its original, balanced-coupling formulation, the model exhibits two phases: a quiescent regime, where all activity ceases, and a regime of ongoing irregular collective activity, termed asynchronous chaos (AC), in which state variables fluctuate strongly in time and across units but average to zero across the network. Building on recent work, we analyze an extension of the SCS model that breaks the positive/negative balance, producing a richer phase diagram. In addition to the classical quiescent and AC phases, two novel regimes emerge, marked by spontaneous symmetry breaking. In the persistent-activity (PA) phase, each unit settles into a distinct, stable activation state. In the synchronous-chaotic (SC) phase, dynamics remain irregular and chaotic but fluctuate around a nonzero mean, generating sustained long-time autocorrelations.

In particular, we define the generalized Sompolinsky–Crisanti–Sommers (SCS) model [2] by considering a large recurrent network of N neurons. The state of neuron $i \in \{1, \ldots, N\}$ is described by a continuous variable $x_i(t) \in [-1, 1]$, whose dynamics evolves according to the set of differential equations:

$$\partial_t x_i(t) = -x_i(t) + \phi\left(\sum_{j=1}^{N} W_{ij} x_j(t)\right) \tag{1}$$

where $\phi(x)$ is a nonlinear gain function, here chosen to be $\tanh(x)$, and W_{ij} are quenched synaptic weights drawn independently from a Gaussian ensemble with mean $\overline{W_{ij}} = J_0/N$ and variance $\overline{W_{ij}^2} - (J_0/N)^2 = J^2/N$, where $\bar{\cdot}$ denotes the average over quenched disorder. The phase diagram is characterized with two order parameters: the mean activity $\hat{M}$ and the second moment $\hat{q}$. Both parameters can be computed in the microscopic picture:

$$\hat{M} = \frac{1}{N}\sum_i \frac{1}{T}\int_0^T dt\, x_i(t) \; ; \; \hat{q} = \frac{1}{N}\sum_i \frac{1}{T}\int_0^T dt\, x_i^2(t) \; ; \; \hat{C}(\tau) = \frac{1}{N}\sum_i \frac{1}{T}\int_0^T dt\, x_i(t) x_i(t+\tau) \tag{2}$$

averaged over the quenched disordered matrix W and for large enough number of units N and time window T.

In the thermodynamic limit $N \to \infty$, the system is assumed to be self-averaging and can therefore be described by a representative neuron subject to an effective stochastic process.

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In this way, following the dynamical mean-field (DMF) framework, originally introduced in [1] and later extended via path-integral method the dynamics of this representative unit are governed by the following stochastic DMF equation :

$$\partial_t x(t) = -x(t) + \phi\left(gJ_0 M(t) + gJ\,\eta(t)\right) \tag{3}$$

where $\eta(t)$ is a Gaussian process with $\langle \eta(t) \rangle = 0$ and $\langle \eta(t)\eta(s) \rangle = C(t, s)$, encoding the influence of the network on a single focus unit and the mean activity $M(t)$ and autocorrelation $C(t, s)$ are fixed by imposing self-consistency:

$$M(t) = \langle x(t) \rangle, \qquad C(t, s) = \langle x(t)x(s) \rangle, \tag{4}$$

with averages $\langle \cdot \rangle$ taken over realizations of the effective Gaussian noise, $\eta(t)$.

Based on this dynamical mean-field formalism, complemented by extensive numerical simulations, we show how structural disorder gives rise to symmetry and ergodicity breaking. Remarkably, the resulting phase diagram closely mirrors that of the Sherrington–Kirkpatrick spin-glass model, with the onset of the SC phase coinciding with the transition associated with replica-symmetry breaking. All key features of spin glasses, including ergodicity breaking, have clear counterparts in this recurrent network context, highlighting a unified perspective on complexity in disordered systems. These findings provide a theoretical foundation for understanding how high-dimensional chaos and ergodicity breaking can support robust, flexible computation in recurrent networks, offering insights relevant to both biological and artificial systems.

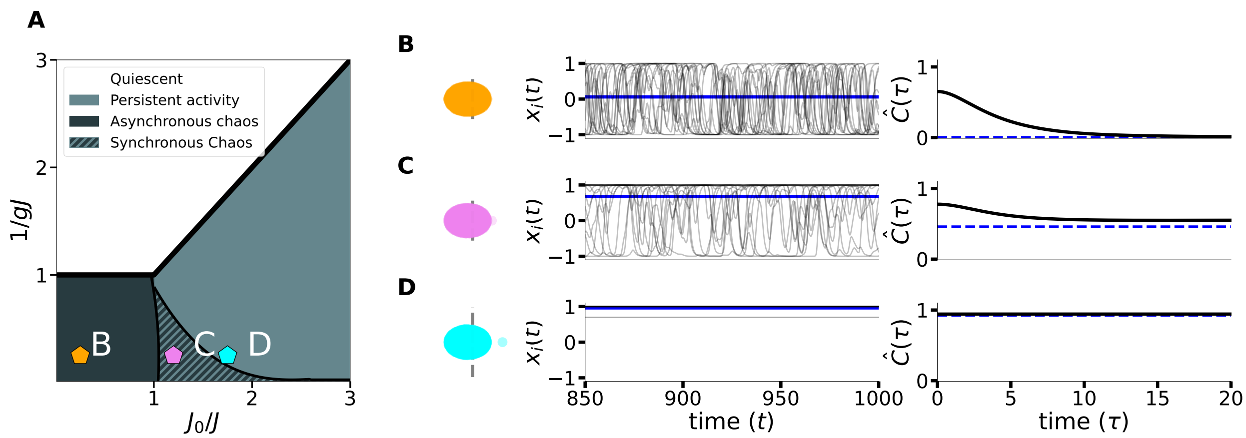

FIG. 1. Phase diagram and representative trajectories of the random recurrent network model. **Panel A**: Phase diagram in the $(J_0/J, 1/gJ)$ plane, from DMF analyses and confirmed by simulations. Four dynamical regimes are shown with their order parameters: quiescent (Q) $\hat{M} = 0$, $\hat{q} = 0$; persistent activity (PA) $\hat{M} \neq 0$, $\hat{q} = 0$; asynchronous chaos (AC) $\hat{M} = 0$, $\hat{q} \neq 0$, with vanishing long-lag correlations; and synchronous chaos (SC) $\hat{M} \neq 0$, $\hat{q} \neq 0$, with persistent correlations. Markers indicate the parameter choices for panels (B)–(D). **Panels B-C**: Representative trajectories from simulations of the microscopic dynamics, Eq.(1), with $N = 2000$ neurons. For each regime, we show: the eigenvalue spectrum of the synaptic matrix; the time evolution of individual unit dynamics $x_i(t)$ along with the population mean $\hat{M}$ (blue line); and the autocorrelation function $\hat{C}(\tau)$ as a function of lag time τ along with $\hat{M}^2$ (dashed, blue line).

[1] Sompolinsky H, Crisanti A and Sommers H J, *Chaos in Random Neural Networks* (1988 Physical Review Letters).

[2] Carles Martorell, Rubén Calvo, Alessia Annibale, Miguel A. Muñoz, *Dynamically selected steady states and criticality in non-reciprocal networks*, (Chaos, Solitons and Fractals 2024).