Anticipated synchronization in stochastic individual-based models

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Anticipated Synchronization (AS) describes situations in which a system synchronises with the future state of another [1]. Thus, the driven system 'forecasts' the state of the driver. Notably, this forecast occurs without any outside intervention. Common setups include two identical copies of a system, coupled via a delayed interaction. AS has been reported in chaotic, excitable and/or spatially extended systems [1, 2, 3]. In these previous works, they used deterministic systems with or without noise.

In this work, we extend AS to agent-based models. The typical setup used to obtain AS in previous works can be described by the following equation:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}[\mathbf{x}(t)] \\ \dot{\mathbf{y}}(t) = \mathbf{f}[\mathbf{y}(t)] + K\left[\mathbf{x}(t) - \mathbf{y}(t-\tau)\right] \end{cases} \tag{1}$$

where the variable $\mathbf{x}$ is for the driver system, the variable $\mathbf{y}$ is for the driven system, $g(\tau)$ is the distribution of the delay, and $\mathbf{f}$ is a given function that determines the dynamical system to be investigated.

In a similar fashion, we will create to identical copies of an agent-based model. We will not modify the dynamics of the driver system, but we will add the following reactions:

$$X_i \quad \overset{K_i}{\rightarrow} \quad X_i + Y_i, \tag{2}$$

$$Y_i \quad \underset{\tau}{\overset{K_i}{\Rightarrow}} \quad \emptyset, \tag{3}$$

where X_i are particles of the driver system, Y_i are the particles of the driven system, and i goes over the number of species of each of the systems.

We will consider two different scenarios: one if which all the delays reactions that are triggered get executed (uninterrupted model) and another one in which a delayed reaction can be interrupted by another reaction (interrupted model).

The model in which every delayed reaction is executed has as a deterministic limit the setup explained above. However, a delayed reaction could potentially try to remove a particle of type Y_i when there are no more particle of this species because they have disappeared through other reactions. In order to fix this problem, we studied a model in which the delayed reactions can be interrupted. This new model guarantees that when a delayed reaction is executed there is at least one particle to remove. However, its behavior is different.

We studied the Brusselator, which is a two dimensional model for chemical reactions that has two regions of our interest. A region in which the deterministic system exhibits periodic behavior and a region in which it just goes to a fixed point.

In the region with periodic behavior for the deterministic model we checked that the agent-based model trajectories are basically the deterministic trajectories plus noise. This means that for the uninterrupted model we find a region of values for K_i and τ in which there is AS. For the interrupted model we also find synchronization, but the anticipation behaves different for the different variables: mostly one exhibits AS and the other one exhibits delayed synchronization.

In the other region in which the deterministic system goes to a fixed point the agent-based models show noise-induced oscillations. In addition, in this case we can carry out the Linear Noise Approximation in order to obtain analytical results. Both simulations and theory agrees quite well. We find that the behavior in this region is similar to the one with deterministic oscillations. So the uninterrupted model exhibits AS with noise-induced oscillation, while the interrupted model exhibits the same type of synchronization explained before.

Our findings extend the concept of anticipated synchronization. It shows that it is still possible to observe AS for noise-induced oscillations. Similar tools could be used in order to understand AS in other noise-induced phenomena in agent-based models. For instance, one could investigate AS in systems that show bistability.

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